Harold's Business Calculus
Cheat Sheet
29 June 2018

Algebra	
Exponents	
1. Multiplication	$a^n a^m = a^{n+m}$
2. Power to a Power	$(a^n)^m = a^{nm}$
3. Zero Power	$a^0 = 1 \quad if\ a \neq 0$
4. Power Sign Change	$a^{-n} = \frac{1}{a^n} \quad and \quad \frac{1}{a^{-n}} = a^n$
Radicals	
5. Convert to Power	$\sqrt[n]{a} = a^{\frac{1}{n}}$
Logarithms	
6. Definition	$x = b^y \quad \equiv \quad y = \log_b x$
7. Powers	$\log_b x^r = r \log_b x$ $ln\, x^r = r\, ln\, x$
8. Multiplication	$ln(xy) = ln\, x + ln\, y$
9. Division	$ln\left(\frac{x}{y}\right) = ln\, x - ln\, y$

Derivatives	(Map to Larson's 1-pager of common derivatives)
0. Chain Rule	$\frac{d}{dx}[f \circ g(x)] = \frac{d}{dx}\left[f\big(g(x)\big)\right] = f'\big(g(x)\big)g'(x)$
1. Constant Multiple Rule	$\frac{d}{dx}[cf(x)] = cf'(x)$
2. Sum and Difference Rule	$\frac{d}{dx}[f \pm g] = f' \pm g'$
3. Product Rule	$\frac{d}{dx}[fg] = fg' + g\,f'$
4. Quotient Rule	$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$ $(think\ \frac{f}{g} = fg^{-1}\ then\ apply\ rule\ \#3)$
5. Constant Rule	$\frac{d}{dx}[c] = 0$ $(think\ \ \mathrm{c} = \mathrm{c}x^0 \rightarrow c0x^{-1} = 0\ \ after\ applying\ rule\ \#6)$
6. Power Rule	$\frac{d}{dx}[cx^n] = cnx^{n-1}$
General Power Rule	$\frac{d}{dx}[f^n] = nf^{n-1}f'$
7. Power Rule for x	$\frac{d}{dx}[x] = 1$ $(think\ \ x = x^1 \rightarrow 1x^0 = 1\ \ after\ applying\ rule\ \#6)$

9. Natural Logorithm Rule	$\frac{d}{dx}[\ln x] = \frac{1}{x}$
10. Natural Exponential Rule	$\frac{d}{dx}[e^x] = e^x$

Integrals	
1. Power Rule	$\int cx^n\,dx = c\frac{x^{n+1}}{n+1} + C$
2. Natural Exponential Rule	$\int ce^f\,dx = c\frac{e^f}{f'} + C$
3. U-Substitution	$\int f(x)\,dx = F(x) + C$ $u = \underline{\text{ part of } f(x) }$ $du = \_\_\_\_ \, dx$

Critical Points	
First Derivative (Slope Formula)	$f'(x) = 0$ *finds critical points (min. or max.)* *Don't forget to check the boundaries: f(a) and f(b)*
Second Deriviative (Test for Concavity)	If $f''(x) \rightarrow +$, then cup up $\cup$ (min.) If $f''(x) \rightarrow -$, then cup down $\cap$ (max.)

Tangent Lines	
Genreal Form	$ax + by + c = 0$
Slope-Intercept Form	$y = mx + b$
Point-Slope Form	$y - y_0 = m(x - x_0)$ $where\ m = f'(x_0)\ at\ point\ (x_0, y_0)$
Calculus Form	$y = f'(c)(x - c) + f(c)$
Slope	$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dy}{dx} = f'(x)$

Implicit Differentiation	Example
$\frac{d}{dx}[cx^n y^m = c]$	$c[(x^n)(y^m)' + (y^m)(x^n)'] = 0$ $c[(x^n)(my^{m-1}y') + (y^m)(nx^{n-1}x')] = 0$ $cmx^n y^{m-1}y' + cny^m x^{n-1} = 0\ \ since\ x' = 1$ $cmx^n y^{m-1}y' = -cnx^{n-1}y^m$ $y' = -\frac{cnx^{n-1}y^m}{cmx^n y^{m-1}}$ $y' = -\frac{nx^{n-1}y^m}{mx^n y^{m-1}}$ $y' = -\frac{n}{m}x^{n-1-n}y^{m-(m-1)}$ $y' = -\frac{n}{m}x^{-1}y^1$ $y' = -\frac{ny}{mx}$