## Algebra

<table>
<thead>
<tr>
<th>Exponents</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Multiplication</td>
<td>$a^n a^m = a^{n+m}$</td>
</tr>
<tr>
<td>2. Power to a Power</td>
<td>$(a^n)^m = a^{nm}$</td>
</tr>
<tr>
<td>3. Zero Power</td>
<td>$a^0 = 1$ if $a \neq 0$</td>
</tr>
<tr>
<td>4. Power Sign Change</td>
<td>$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Radicals</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Convert to Power</td>
<td>$\sqrt[n]{a} = a^{\frac{1}{n}}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Logarithms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6. Definition</td>
<td>$x = b^y \equiv y = \log_b x$</td>
</tr>
</tbody>
</table>
| 7. Powers | \[
\log_b x^r = r \log_b x \\
\ln x^r = r \ln x
\] |
| 8. Multiplication | $\ln(xy) = \ln x + \ln y$ |
| 9. Division | $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$ |

## Derivatives

(Map to Larson's 1-pager of common derivatives)

<table>
<thead>
<tr>
<th>Derivatives</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0. Chain Rule</td>
<td>$\frac{d}{dx}[f \circ g(x)] = \frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$</td>
</tr>
<tr>
<td>1. Constant Multiple Rule</td>
<td>$\frac{d}{dx}[cf(x)] = cf'(x)$</td>
</tr>
<tr>
<td>2. Sum and Difference Rule</td>
<td>$\frac{d}{dx}[f \pm g] = f' \pm g'$</td>
</tr>
<tr>
<td>3. Product Rule</td>
<td>$\frac{d}{dx}[fg] = fg' + gf'$</td>
</tr>
<tr>
<td>4. Quotient Rule</td>
<td>$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$</td>
</tr>
<tr>
<td>(think $\frac{f}{g} = fg^{-1}$ then apply rule #3)</td>
<td></td>
</tr>
<tr>
<td>5. Constant Rule</td>
<td>$\frac{d}{dx}[c] = 0$</td>
</tr>
<tr>
<td>(think $c = cx^0 \rightarrow c0x^{-1} = 0$ after applying rule #6)</td>
<td></td>
</tr>
<tr>
<td>6. Power Rule</td>
<td>$\frac{d}{dx}[cx^n] = cnx^{n-1}$</td>
</tr>
<tr>
<td>General Power Rule</td>
<td>$\frac{d}{dx}[f^n] = nf^{n-1}f'$</td>
</tr>
<tr>
<td>7. Power Rule for x</td>
<td>$\frac{d}{dx}[x] = 1$</td>
</tr>
<tr>
<td>(think $x = x^1 \rightarrow 1x^0 = 1$ after applying rule #6)</td>
<td></td>
</tr>
</tbody>
</table>
9. **Natural Logarithm Rule**
\[
\frac{d}{dx} \ln x = \frac{1}{x}
\]

10. **Natural Exponential Rule**
\[
\frac{d}{dx} e^x = e^x
\]

### Integrals
| 1. **Power Rule** | \[
\int cx^n \, dx = c \frac{x^{n+1}}{n+1} + C
\] |
| 2. **Natural Exponential Rule** | \[
\int e^x \, dx = e^x + C
\] |
| 3. **U-Substitution** | \[
\int f(x) \, dx = F(x) + C
\]

\[u = \text{part of } f(x) \quad du = \text{_____} \, dx\]

### Critical Points
- **First Derivative**
  - \(f'(x) = 0\) finds critical points (min. or max.)
  - Don’t forget to check the boundaries: \(f(a)\) and \(f(b)\)
- **Second Derivative**
  - If \(f''(x) \to +\), then cup up \(\cup\) (min.)
  - If \(f''(x) \to -\), then cup down \(\bigcap\) (max.)

### Tangent Lines
| General Form | \(ax + by + c = 0\) |
| Slope-Intercept Form | \(y = mx + b\) |
| Point-Slope Form | \(y - y_0 = m(x - x_0)\) where \(m = f'(x_0)\) at point \((x_0, y_0)\) |
| Calculus Form | \(y = f'(c)(x - c) + f(c)\) |

### Implicit Differentiation
| Example | \[
c[(x^n)(y^m)'] + (y^m)(x^n)' = 0
\]
| | \[
c[(x^n)(my^{m-1}y') + (y^m)(nx^{n-1}x')] = 0
\]
| | \[
 cmb^n y^{m-1} y' + cny^m x^{n-1} = 0 \text{ since } x' = 1
\]
| | \[
cmx^n y^{m-1} y' = -cmx^{n-1} y^m
\]
| | \[
y' = - \frac{cmx^n y^m}{cmx^n y^{m-1}}
\]
| | \[
y' = - \frac{nx^{n-1} y^m}{mx^n y^{m-1}}
\]
| | \[
y' = - \frac{n}{m} x^{n-1} y^{m-(m-1)}
\]
| | \[
y' = - \frac{n}{m} y m^{-1} y^1
\]
| | \[
y' = - \frac{ny}{mx}
\]

\[
\frac{d}{dx} [cx^n y^m = c]
\]