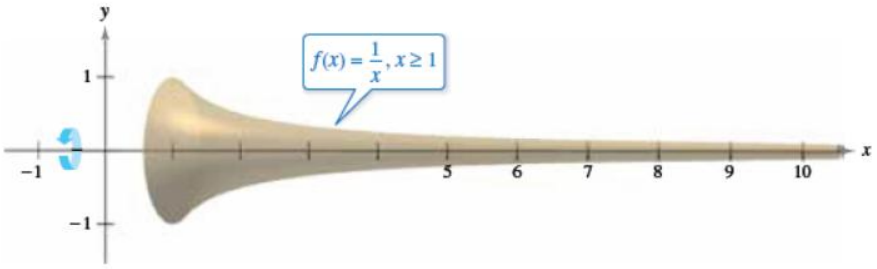
Harold's Math is Fun – Part 2
Cheat Sheet
23 March 2026

Deceptive Calculus Proofs

<table>
<tr><th>Description</th><th>Equation</th><th>Translation</th></tr>
<tr><td rowspan="5">Gabriel's Horn</td><td>$f(x) = \frac{1}{x}, x \geq 1$</td><td>Has a finite volume and an infinite suface area.</td></tr>
<tr><td colspan="2"> 
</td></tr>
<tr><td>Volume</td><td>Surface Area</td></tr>
<tr><td>$V = \pi \int_1^a \left(\frac{1}{x}\right)^2 dx = \pi\left(1 - \frac{1}{a}\right)$
$\lim_{a\to\infty} V = \lim_{a\to\infty} \pi\left(1 - \frac{1}{a}\right) = \pi$</td><td>$A = 2\pi \int_1^a \frac{1}{x}\sqrt{1 + \left(-\frac{1}{x^2}\right)^2}\, dx$
$> 2\pi \int_1^a \frac{1}{x} dx = 2\pi \ln a$
$\lim_{a\to\infty} A > \lim_{a\to\infty} 2\pi \ln a = \infty$</td></tr>
<tr><td colspan="2">The mismatch comes from how slowly $\frac{1}{x}$ decays.</td></tr>
<tr><td>Log Cabin</td><td>$\int \frac{d\,Cabin}{Cabin} = ln(Cabin) + C$</td><td>Log cabin by the sea.</td></tr>
</table>

Natural Number (e)

Description	Equation	Translation

Pi (π)

Description	Equation	Translation
Dessert	$(AP_{\pi})^{t}$	A piece of pie raised to the teeth.

Sources

- Wikipedia (2026). Gabriel's horn. https://en.wikipedia.org/wiki/Gabriel%27s_horn
- Microsoft Copilot (2026). "Deceptive Calculus Proofs like Gabriels' horn and others.", https://copilot.microsoft.com/chats/L4fpub6DLCh9zuuj3cmue

