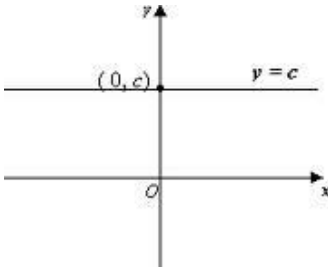
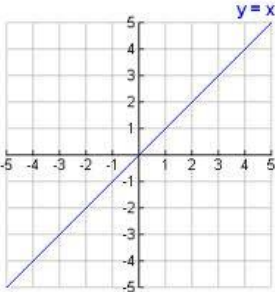
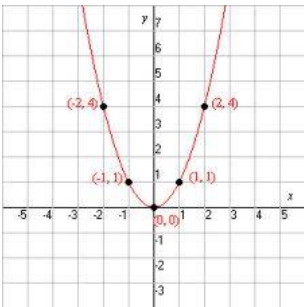
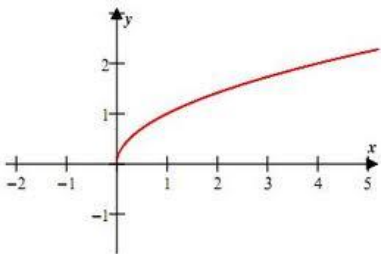


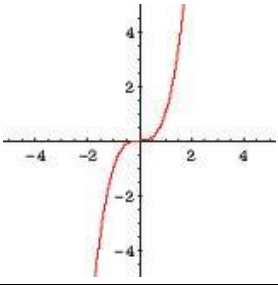
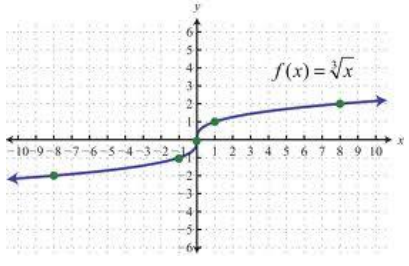
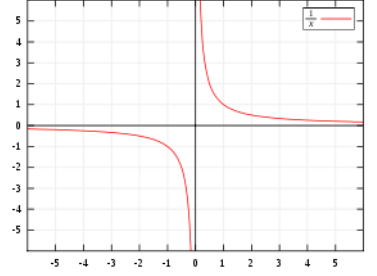
## Harold's Parent Functions "Cheat Sheet"

6 November 2019

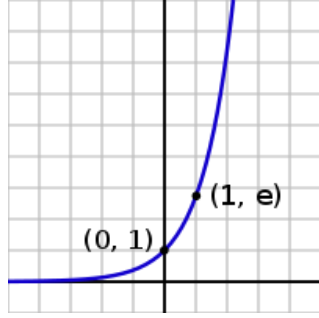
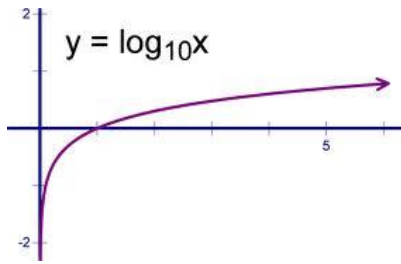
Function Name	Parent Function	Graph	Characteristics
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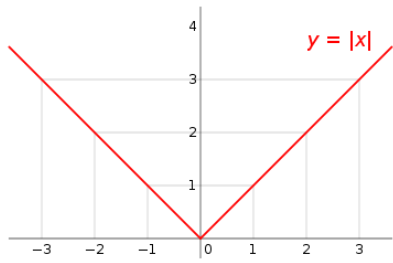
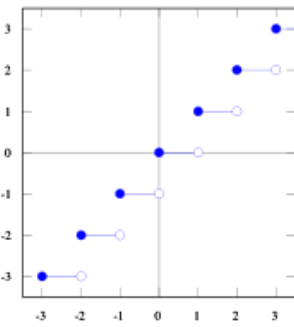
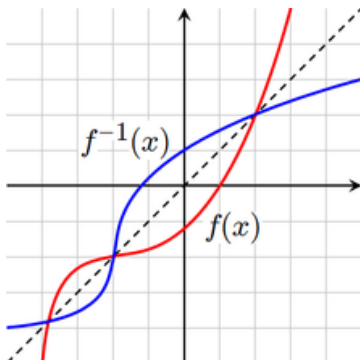
### Algebra

<b>Constant</b>	$f(x) = c$		Domain: $(-\infty, \infty)$ Range: $[c, c]$ Inverse Function: Undefined (asymptote) Restrictions: $c$ is a real number Odd/Even: Even General Form: $Ay + B = 0$
<b>Linear or Identity</b>	$f(x) = x$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: Same as parent Restrictions: $m \neq 0$ Odd/Even: Odd General Forms: $Ax + By + C = 0$ $y = mx + b$ $y - y_0 = m(x - x_0)$
<b>Quadratic or Square</b>	$f(x) = x^2$		Domain: $(-\infty, \infty)$ Range: $[0, \infty)$ Inverse Function: $f^{-1}(x) = \sqrt{x}$ Restrictions: None Odd/Even: Even General Form: $Ax^2 + By + Cx + D = 0$
<b>Square Root</b>	$f(x) = \sqrt{x}$		Domain: $[0, \infty)$ Range: $[0, \infty)$ Inverse Function: $f^{-1}(x) = x^2$ Restrictions: $x \geq 0$ Odd/Even: Neither General Form: $f(x) = a\sqrt{b(x-h)} + k$

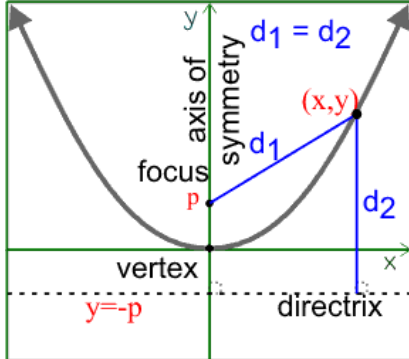
Function Name	Parent Function	Graph	Characteristics
<b>Cubic</b>	$f(x) = x^3$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = \sqrt[3]{x}$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a(b(x-h))^3 + k$
<b>Cube Root</b>	$f(x) = \sqrt[3]{x}$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = x^3$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a\sqrt[3]{b(x-h)} + k$
<b>Reciprocal or Rational</b>	$f(x) = \frac{1}{x}$		Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$ Inverse Function: Same as parent Restrictions: $x \neq 0$ Odd/Even: Odd General Form: $f(x) = \frac{a}{b(x-h)} + k$

### Transcendentals

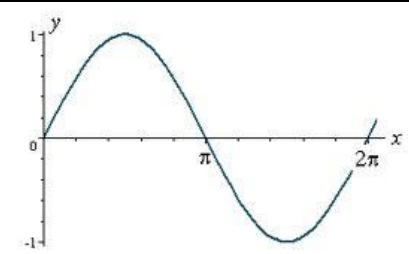
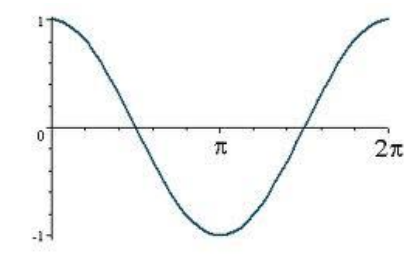
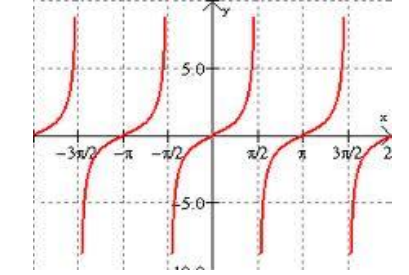
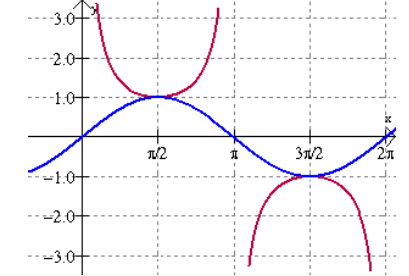
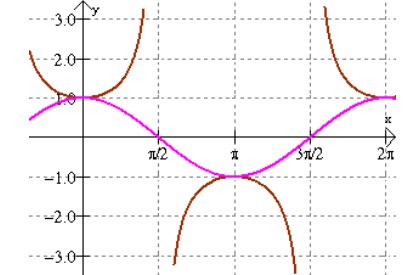
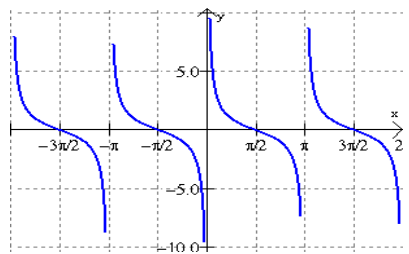
<b>Exponential</b>	$f(x) = 10^x$ or $f(x) = e^x$		Domain: $(-\infty, \infty)$ Range: $(0, \infty)$ Inverse Function: $f^{-1}(x) = \log x$ or $f^{-1}(x) = \ln x$ Restrictions: None, $x$ can be complex Odd/Even: Neither General Form: $f(x) = a 10^{(b(x-h))} + k$
<b>Logarithmic</b>	$f(x) = \log x$ or $f(x) = \ln x$		Domain: $(0, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = 10^x$ or $f^{-1}(x) = e^x$ Restrictions: $x > 0$ Odd/Even: Neither General Form: $f(x) = a \log(b(x-h)) + k$

Function Name	Parent Function	Graph	Characteristics
<b>Absolute Value</b>	$f(x) =  x $		Domain: $(-\infty, \infty)$ Range: $[0, \infty)$ Inverse Function: $f^{-1}(x) = x$ for $x \geq 0$ Restrictions: $f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$ Odd/Even: Even General Form: $f(x) = a b(x - h)  + k$
<b>Greatest Integer or Floor</b>	$f(x) = [x]$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ whole numbers only Inverse Function: Undefined (asymptotic) Restrictions: Real numbers only Odd/Even: Neither General Form: $f(x) = a[b(x - h)] + k$
<b>Inverse Functions</b>	$f(f^{-1}(x)) = x$ $f^{-1}(f(x)) = x$		Domain of $x \rightarrow$ Domain of $y$ Range of $y \rightarrow$ Range of $x$ Inverse Function: By definition Restrictions: None Odd/Even: Odd General Form: $f(x) = a f(b(x - h)) + k$ Algebraically: Swap $x \leftrightarrow y$ , then solve for $y$ Graphically: Rotate about $45^\circ$ line $y = x$

### Conic Sections

<b>Parabola</b>	$y = ax^2$		Domain: $(-\infty, \infty)$ Range: $[k, \infty)$ or $(-\infty, k]$ Inverse Function: $f^{-1}(x) = \sqrt{x}$ Restrictions: None Odd/Even: Even Vertex: $(h, k)$ Focus: $(h, k + p)$ General Forms: $(x - h)^2 = 4p(y - k)$ $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where $B^2 - 4AC = 0$
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Function Name	Parent Function	Graph	Characteristics
Circle	$x^2 + y^2 = r^2$		Domain: $[-r + h, r + h]$ Range: $[-r + k, r + k]$ Inverse Function: Same as parent Restrictions: None Odd/Even: Both Focus : $(h, k)$ General Forms: $(x - h)^2 + (y - k)^2 = r^2$ $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ <i>where <math>A = C</math> and <math>B = 0</math></i>
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$		Domain: $[-a + h, a + h]$ Range: $[-b + k, b + k]$ Inverse Function: $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ Restrictions: None Odd/Even: Both Foci : $c^2 = a^2 - b^2$ General Forms: $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$ $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ <i>where <math>B^2 - 4AC &lt; 0</math></i>
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$		Domain: $(-\infty, -a+h] \cup [a+h, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ Restrictions: Domain is restricted Odd/Even: Both Foci : $c^2 = a^2 + b^2$ General Forms: $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ <i>where <math>B^2 - 4AC &gt; 0</math></i>

Function Name	Parent Function	Graph	Characteristics
<b>Trigonometry</b>			
<b>Sine</b>	$f(x) = \sin x$		Domain: $(-\infty, \infty)$ Range: $[-1, 1]$ Inverse Function: $f^{-1}(x) = \sin^{-1} x$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a \sin(b(x - h)) + k$
<b>Cosine</b>	$f(x) = \cos x$		Domain: $(-\infty, \infty)$ Range: $[-1, 1]$ Inverse Function: $f^{-1}(x) = \cos^{-1} x$ Restrictions: None Odd/Even: Even General Form: $f(x) = a \cos(b(x - h)) + k$
<b>Tangent</b>	$f(x) = \tan x$ $= \frac{\sin x}{\cos x}$		Domain: $(-\infty, \infty)$ except for $x = \frac{\pi}{2} \pm n\pi$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = \tan^{-1} x$ Restrictions: Asymptotes at $x = \frac{\pi}{2} \pm n\pi$ Odd/Even: Odd General Form: $f(x) = a \tan(b(x - h)) + k$
<b>Cosecant</b>	$f(x) = \csc x$ $= \frac{1}{\sin x}$		Domain: $(-\infty, \infty)$ except for $x = \pm n\pi$ Range: $(-\infty, -1] \cup [1, \infty)$ Inverse Function: $f^{-1}(x) = \csc^{-1} x$ Restrictions: Range is bounded Odd/Even: Odd General Form: $f(x) = a \csc(b(x - h)) + k$
<b>Secant</b>	$f(x) = \sec x$ $= \frac{1}{\cos x}$		Domain: $(-\infty, \infty)$ except for $x = \frac{\pi}{2} \pm n\pi$ Range: $(-\infty, -1] \cup [1, \infty)$ Inverse Function: $f^{-1}(x) = \sec^{-1} x$ Restrictions: Range is bounded Odd/Even: Even General Form: $f(x) = a \sec(b(x - h)) + k$
<b>Cotangent</b>	$f(x) = \cot x$ $= \frac{1}{\tan x}$		Domain: $(-\infty, \infty)$ except for $x = \pm n\pi$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = \cot^{-1} x$ Restrictions: Asymptotes at $x = \pm n\pi$ Odd/Even: Odd General Form: $f(x) = a \cot(b(x - h)) + k$

Function Name	Parent Function	Graph	Characteristics
<b>Arcsine</b>	$f(x) = \sin^{-1} x$		Domain: $[-1, 1]$ Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$ or Quadrants I & IV Inverse Function: $f^{-1}(x) = \sin x$ Restrictions: Range & Domain are bounded Odd/Even: Odd General Form: $f(x) = a \sin^{-1}(b(x-h)) + k$
<b>Arccosine</b>	$f(x) = \cos^{-1} x$		Domain: $[-1, 1]$ Range: $[0, \pi]$ or Quadrants I & II Inverse Function: $f^{-1}(x) = \cos x$ Restrictions: Range & Domain are bounded Odd/Even: None General Form: $f(x) = a \cos^{-1}(b(x-h)) + k$
<b>Arctangent</b>	$f(x) = \tan^{-1} x$		Domain: $(-\infty, \infty)$ Range: $(-\frac{\pi}{2}, \frac{\pi}{2})$ or Quadrants I & IV Inverse Function: $f^{-1}(x) = \tan x$ Restrictions: Range is bounded Odd/Even: Odd General Form: $f(x) = a \tan^{-1}(b(x-h)) + k$
<b>Arccosecant</b>	$f(x) = \csc^{-1} x$		Domain: $(-\infty, -1] \cup [1, \infty)$ Range: $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$ or Quadrants I & IV Inverse Function: $f^{-1}(x) = \csc x$ Restrictions: Range & Domain are bounded Odd/Even: Odd General Form: $f(x) = a \csc^{-1}(b(x-h)) + k$
<b>Arcsecant</b>	$f(x) = \sec^{-1} x$		Domain: $(-\infty, -1] \cup [1, \infty)$ Range: $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$ or Quadrants I & II Inverse Function: $f^{-1}(x) = \sec x$ Restrictions: Range & Domain are bounded Odd/Even: Neither General Form: $f(x) = a \sec^{-1}(b(x-h)) + k$
<b>Arccotangent</b>	$f(x) = \cot^{-1} x$		Domain: $(-\infty, \infty)$ Range: $(0, \pi)$ or Quadrants I & II Inverse Function: $f^{-1}(x) = \cot x$ Restrictions: Range is bounded Odd/Even: Neither General Form: $f(x) = a \cot^{-1}(b(x-h)) + k$

Function Name	Parent Function	Graph	Characteristics
<b>Hyperbolics</b>			
<b>Hyperbolic Sine</b>	$f(x) = \sinh x$ $= \frac{e^x - e^{-x}}{2}$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = \sinh^{-1} x$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a \sinh (b(x - h)) + k$
<b>Hyperbolic Cosine</b>	$f(x) = \cosh x$ $= \frac{e^x + e^{-x}}{2}$		Domain: $(-\infty, \infty)$ Range: $[1, \infty)$ Inverse Function: $f^{-1}(x) = \cosh^{-1} x$ Restrictions: None Odd/Even: Even General Form: $f(x) = a \cosh (b(x - h)) + k$
<b>Hyperbolic Tangent</b>	$f(x) = \tanh x$ $= \frac{e^{2x} - 1}{e^{2x} + 1}$		Domain: $(-\infty, \infty)$ Range: $(-1, 1)$ Inverse Function: $f^{-1}(x) = \tanh^{-1} x$ Restrictions: Asymptotes at $y = \pm 1$ Odd/Even: Odd General Form: $f(x) = a \tanh (b(x - h)) + k$
<b>Hyperbolic Cosecant</b>	$f(x) = \operatorname{csch} x$ $= \frac{1}{\sinh x}$		Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup [0, \infty)$ Inverse Function: $f^{-1}(x) = \operatorname{csch}^{-1} x$ Restrictions: Asymptotes at $x = 0, y = 0$ Odd/Even: Odd General Form: $f(x) = a \operatorname{csch} (b(x - h)) + k$
<b>Hyperbolic Secant</b>	$f(x) = \operatorname{sech} x$ $= \frac{1}{\cosh x}$		Domain: $(-\infty, \infty)$ Range: $(0, 1]$ Inverse Function: $f^{-1}(x) = \operatorname{sech}^{-1} x$ Restrictions: Asymptote at $y = 0$ Odd/Even: Even General Form: $f(x) = a \operatorname{sech} (b(x - h)) + k$
<b>Hyperbolic Cotangent</b>	$f(x) = \operatorname{coth} x$ $= \frac{1}{\tanh x}$		Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 1) \cup (1, \infty)$ Inverse Function: $f^{-1}(x) = \operatorname{coth}^{-1} x$ Restrictions: Asymptotes at $x = 0, y = \pm 1$ Odd/Even: Odd General Form: $f(x) = a \operatorname{coth} (b(x - h)) + k$

Function Name	Parent Function	Graph	Characteristics
<b>Hyperbolic Arcsine</b>	$f(x) = \sinh^{-1} x$ $= \ln(x + \sqrt{x^2 + 1})$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = \sinh x$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a \sinh^{-1}(b(x - h)) + k$
<b>Hyperbolic Arccosine</b>	$f(x) = \cosh^{-1} x$ $= \ln(x + \sqrt{x^2 - 1})$		Domain: $[1, \infty)$ Range: $[0, \infty)$ Inverse Function: $f^{-1}(x) = \cosh x$ Restrictions: $y \geq 0$ Odd/Even: Neither General Form: $f(x) = a \cosh^{-1}(b(x - h)) + k$
<b>Hyperbolic Arctangent</b>	$f(x) = \tanh^{-1} x$ $= \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$		Domain: $(-1, 1)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = \tanh x$ Restrictions: Asymptotes at $x = \pm 1$ Odd/Even: Odd General Form: $f(x) = a \tanh^{-1}(b(x - h)) + k$
<b>Hyperbolic Arccosecant</b>	$f(x) = \operatorname{csch}^{-1} x$ $= \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1}\right)$		Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup [0, \infty)$ Inverse Function: $f^{-1}(x) = \operatorname{csch} x$ Restrictions: Asymptotes at $x = 0, y = 0$ Odd/Even: Odd General Form: $f(x) = a \operatorname{csch}^{-1}(b(x - h)) + k$
<b>Hyperbolic Arcsecant</b>	$f(x) = \operatorname{sech}^{-1} x$ $= \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right)$		Domain: $(0, 1]$ Range: $[0, \infty)$ Inverse Function: $f^{-1}(x) = \operatorname{sech} x$ Restrictions: Odd/Even: Neither General Form: $f(x) = a \operatorname{sech}^{-1}(b(x - h)) + k$
<b>Hyperbolic Arccotangent</b>	$f(x) = \operatorname{coth}^{-1} x$ $= \frac{1}{2} \ln\left(\frac{x+1}{x-1}\right)$		Domain: $[-\infty, -1) \cup (1, \infty]$ Range: $(-\infty, 0) \cup (0, \infty)$ Inverse Function: $f^{-1}(x) = \operatorname{coth} x$ Restrictions: Asymptotes at $x = 0, y = \pm 1$ Odd/Even: Odd General Form: $f(x) = a \operatorname{coth}^{-1}(b(x - h)) + k$



## Graphing Tips

### All Functions

The Seven Function "Levers"	$y = a f(b(x - h)) + k$	Graphing Tips
1) Move up/down $\updownarrow$	k (Vertical translation)	"+" Moves it up
2) Move left/right $\leftrightarrow$	h (Horizontal translation)	"+" Moves it right
3) Stretch up/down $\updownarrow$	a (Vertical dilation)	Larger stretches it taller or makes it grow faster
4) Stretch left/right $\leftrightarrow$	b (Horizontal dilation)	Larger stretches it out wider
5) Flip about x-axis $\cup$	$a \rightarrow -a$	$f(x) \rightarrow -f(x)$ If $f(x) = -f(-x)$ then "odd" function
6) Flip about y-axis $\cup$	$b \rightarrow -b$	$f(x) \rightarrow f(-x)$ If $f(x) = f(-x)$ then "even" function
7) Rotate CW/CCW $\cup$	$\cot 2\theta = \frac{A - C}{B}$	"+" $\theta$ rotates CCW For conic sections, where: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

### Trigonometric Functions

The Six Trig "Levers"	$y = a \sin(b(x - h)) + k$	Graphing Tips	Notes
1) Move up/down $\updownarrow$	k (Vertical translation)	$k = \frac{(\max + \min)}{2}$	If $k = f(x)$ then x-axis is replaced by $f(x)$ -axis
2) Move left/right $\leftrightarrow$	h (Phase shift)	'+' shifts right	$\sin(x) = \cos(x - \pi/2)$
3) Stretch up/down $\updownarrow$	a (Amplitude)	$a = \frac{(\max - \min)}{2}$	a is NOT peak-to-peak on y-axis
4) Stretch left/right $\leftrightarrow$	b (Frequency $\cdot 2\pi$ )	$T = \frac{2\pi}{ b } = \frac{1}{f}$	T = peak-to-peak on $\theta$ -axis $T = \frac{\pi}{b}$ for $\tan(bx)$
5) Flip about y-axis $\cup$	$b \rightarrow -b$	$f(x) \rightarrow f(-x)$	Even Function: $\cos(x) = \cos(-x)$
6) Flip about x-axis $\cup$	$a \rightarrow -a$	$f(x) \rightarrow -f(-x)$	Odd Function: $\sin(x) = -\sin(-x)$