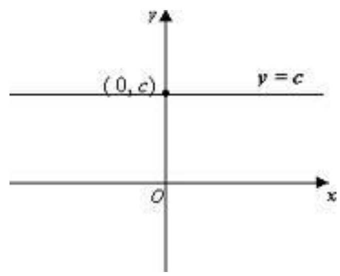
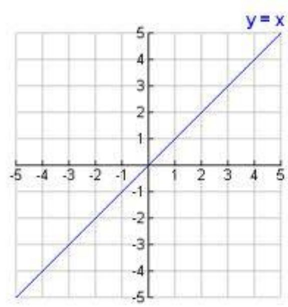
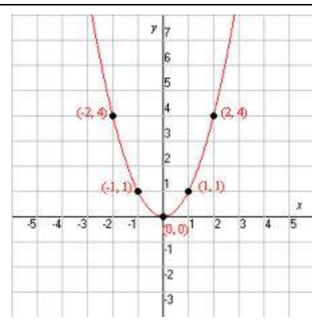
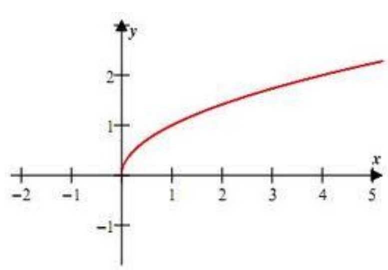
Harold’s Parent Functions
“Cheat Sheet”

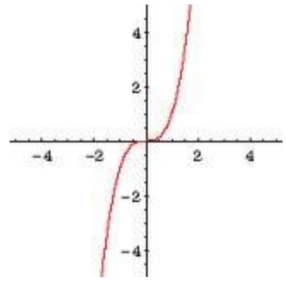
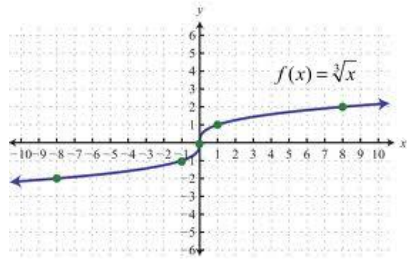
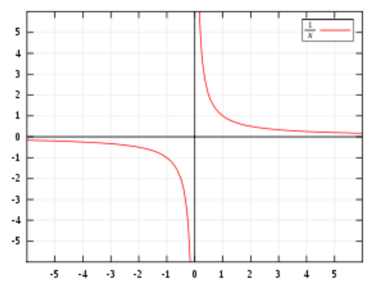
AKA Library of Functions

18 September 2022

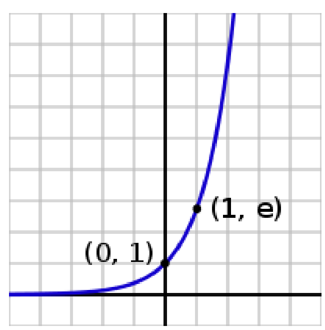
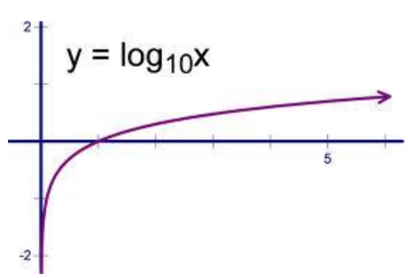
Function Name	Parent Function	Graph	Characteristics

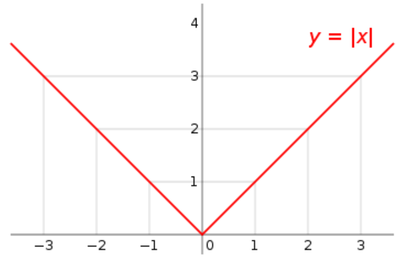
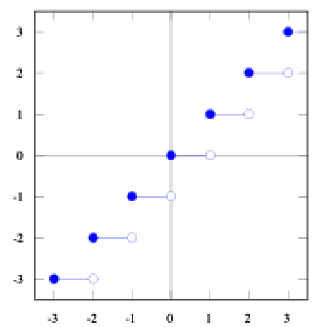
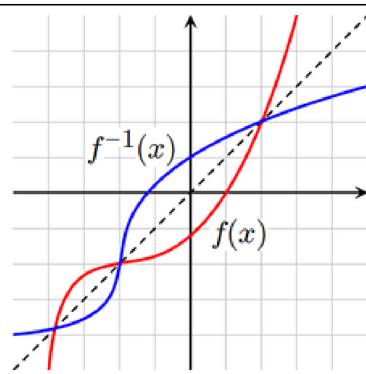
Algebra

Function Name	Parent Function	Graph	Characteristics
Constant	$f(x) = c$		Domain: $(-\infty, \infty)$ Range: [c, c] Inverse Function: Undefined (asymptote) Restrictions: c is a real number Odd/Even: Even General Form: $Ay + B = 0$
Linear **or** **Identity**	$f(x) = x$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: Same as parent Restrictions: m ≠ 0 Odd/Even: Odd General Forms: $Ax + By + C = 0$ $y = mx + b$ $y - y_0 = m(x - x_0)$
Quadratic **or** **Square**	$f(x) = x^2$		Domain: $(-\infty, \infty)$ Range: $[0, \infty)$ Inverse Function: $f^{-1}(x) = \sqrt{x}$ Restrictions: None Odd/Even: Even General Form: $Ax^2 + By + Cx + D = 0$
Square Root	$f(x) = \sqrt{x}$		Domain: $[0, \infty)$ Range: $[0, \infty)$ Inverse Function: $f^{-1}(x) = x^2$ Restrictions: $x \geq 0$ Odd/Even: Neither General Form: $f(x) = a\sqrt{b(x-h)} + k$

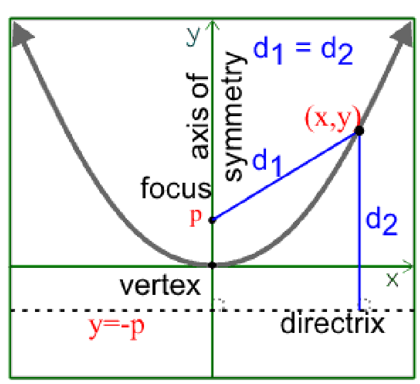
Function Name	Parent Function	Graph	Characteristics
Cubic	$f(x) = x^3$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = \sqrt[3]{x}$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a(b(x-h))^3 + k$
Cube Root	$f(x) = \sqrt[3]{x}$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = x^3$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a\sqrt[3]{b(x-h)} + k$
Reciprocal or Rational	$f(x) = \frac{1}{x}$		Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$ Inverse Function: Same as parent Restrictions: $x \neq 0$ Odd/Even: Odd General Form: $f(x) = \frac{a}{b(x-h)} + k$

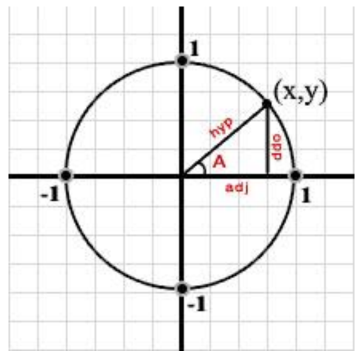
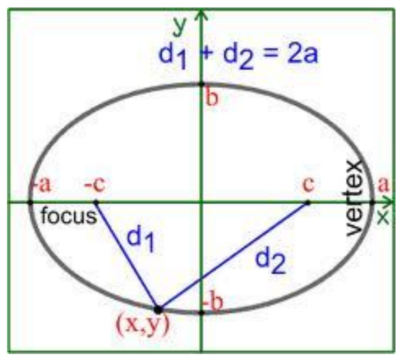
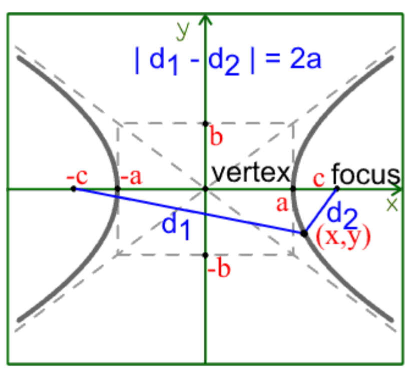
Transcendentals

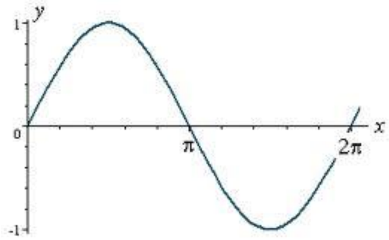
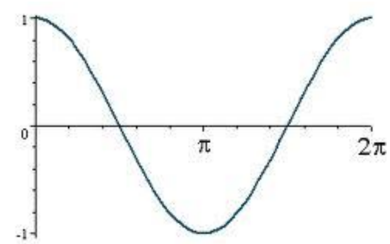
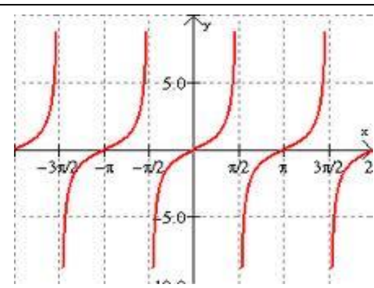
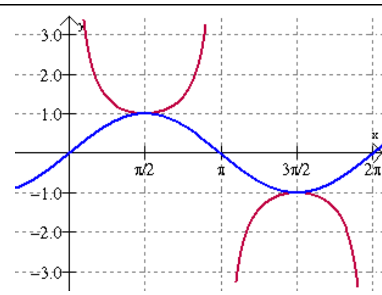
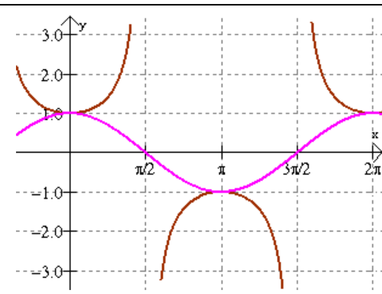
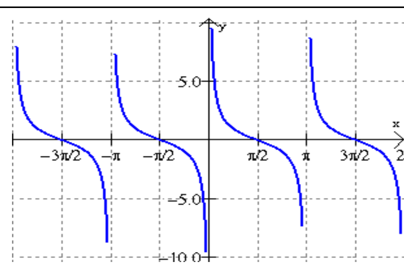
Exponential	$f(x) = 10^x$ *or* $f(x) = e^x$		Domain: $(-\infty, \infty)$ Range: $(0, \infty)$ Inverse Function: $f^{-1}(x) = \log x$ *or* $f^{-1}(x) = \ln x$ Restrictions: None, x can be complex Odd/Even: Neither General Form: $f(x) = a\, 10^{(b(x-h))} + k$
Logarithmic	$f(x) = \log x$ *or* $f(x) = \ln x$		Domain: $(0, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = 10^x$ *or* $f^{-1}(x) = e^x$ Restrictions: $x > 0$ Odd/Even: Neither General Form: $f(x) = a\log(b(x-h)) + k$

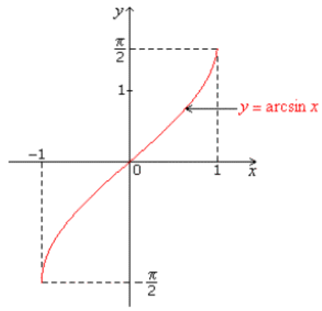
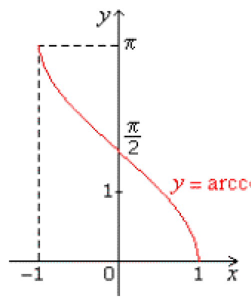
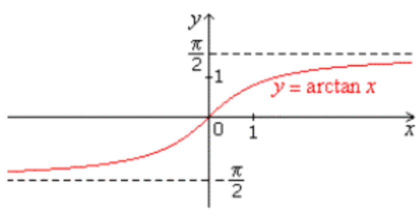
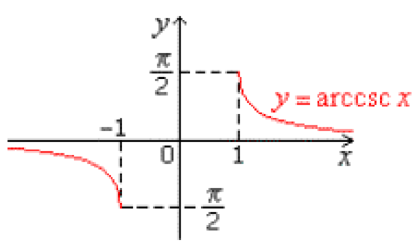
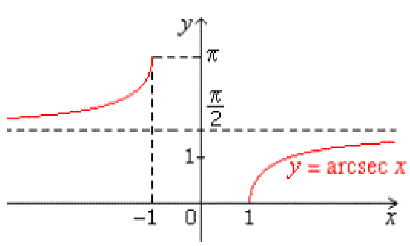
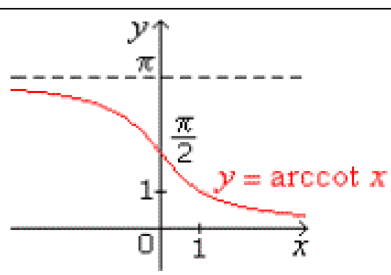
Function Name	Parent Function	Graph	Characteristics
Absolute Value	$f(x) = \|x\|$		Domain: $(-\infty, \infty)$ Range: $[0, \infty)$ Inverse Function: $f^{-1}(x) = x \; for \; x \geq 0$ Restrictions: $f(x) = \begin{cases} x, & if \; x \geq 0 \\ -x, & if \; x < 0 \end{cases}$ Odd/Even: Even General Form: $f(x) = a\|b(x-h)\| + k$
Greatest Integer or Floor	$f(x) = [x]$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ whole numbers only Inverse Function: Undefined (asymptotic) Restrictions: Real numbers only Odd/Even: Neither General Form: $f(x) = a[b(x-h)] + k$
Inverse Functions	$f(f^{-1}(x)) = x$ $f^{-1}(f(x)) = x$		Domain of x → Domain of y Range of y → Range of x Inverse Function: By definition Restrictions: None Odd/Even: Odd General Form: $f(x) = a\,f(b(x-h)) + k$ Algebraically: Swap $x \leftrightarrow y$, then solve for y Graphically: Rotate about 45⁰ line $y = x$

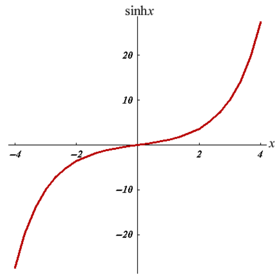
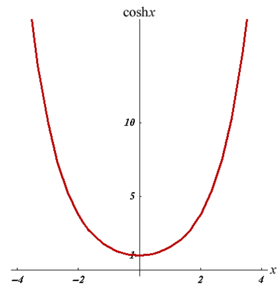
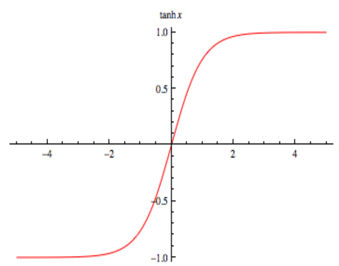
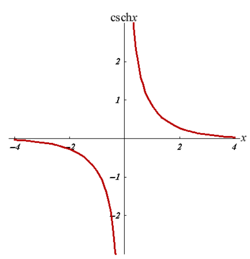
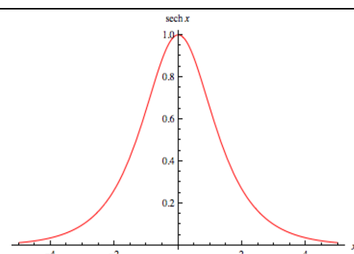
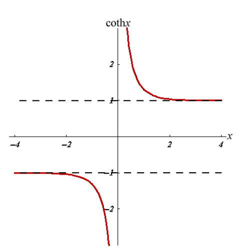
Conic Sections

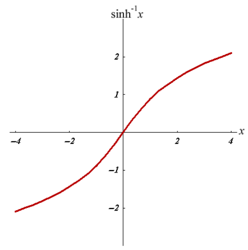
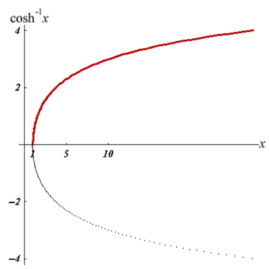
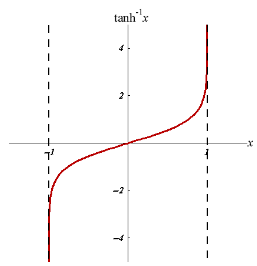
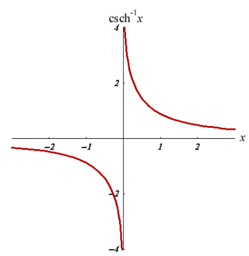
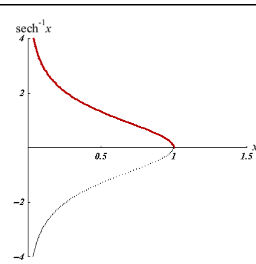
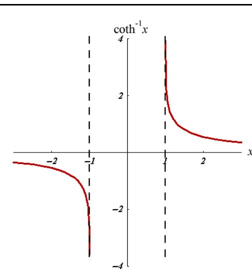
Parabola	$y = ax^2$		Domain: $(-\infty, \infty)$ Range: $[k, \infty)$ or $(-\infty, k]$ Inverse Function: $f^{-1}(x) = \sqrt{x}$ Restrictions: None Odd/Even: Even Vertex : (h, k) Focus : $(h, k+p)$ General Forms: $(x-h)^2 = 4p(y-k)$ $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ $where \; B^2 - 4AC = 0$

Function Name	Parent Function	Graph	Characteristics
Circle	$x^2 + y^2 = r^2$		Domain: $[-r+h,\ r+h]$ Range: $[-r+k,\ r+k]$ Inverse Function: Same as parent Restrictions: None Odd/Even: Both Focus : (h, k) General Forms: $(x-h)^2 + (y-k)^2 = r^2$ $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ $where\ A = C\ and\ B = 0$
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$		Domain: $[-a+h,\ a+h]$ Range: $[-b+k,\ b+k]$ Inverse Function: $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ Restrictions: None Odd/Even: Both Foci : $c^2 = a^2 - b^2$ General Forms: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ $where\ B^2 - 4AC < 0$
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$		Domain: $(-\infty, -a+h] \cup [a+h, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ Restrictions: Domain is restricted Odd/Even: Both Foci : $c^2 = a^2 + b^2$ General Forms: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ $where\ B^2 - 4AC > 0$

Function Name	Parent Function	Graph	Characteristics
Trigonometry			
Sine	$f(x) = sin\, x$		Domain: $(-\infty, \infty)$ with T = 2π/|b| Range: $[-1, 1]$ Inverse Function: $f^{-1}(x) = sin^{-1} x$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a\, sin\, \big(b(x-h)\big) + k$
Cosine	$f(x) = cos\, x$		Domain: $(-\infty, \infty)$ with T = 2π/|b| Range: $[-1, 1]$ Inverse Function: $f^{-1}(x) = cos^{-1} x$ Restrictions: None Odd/Even: Even General Form: $f(x) = a\ \ cos\, \big(b(x-h)\big) + k$
Tangent	$f(x) = tan\, x$ $= \dfrac{sin\, x}{cos\, x}$		Domain: $(-\infty, \infty)$ except for $x = \frac{\pi}{2} \pm n\pi$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = tan^{-1} x$ Restrictions: Asymptotes at $x = \frac{\pi}{2} \pm n\pi$ Odd/Even: Odd General Form: $f(x) = a\, tan\, \big(b(x-h)\big) + k$
Cosecant	$f(x) = csc\, x$ $= \dfrac{1}{sin\, x}$		Domain: $(-\infty, \infty)$ except for $x = \pm n\pi$ Range: $(-\infty, -1] \cup [1, \infty)$ Inverse Function: $f^{-1}(x) = csc^{-1} x$ Restrictions: Range is bounded Odd/Even: Odd General Form: $f(x) = a\ \ csc\, \big(b(x-h)\big) + k$
Secant	$f(x) = \sec x$ $= \dfrac{1}{cos\, x}$		Domain: $(-\infty, \infty)$ except for $x = \frac{\pi}{2} \pm n\pi$ Range: $(-\infty, -1] \cup [1, \infty)$ Inverse Function: $f^{-1}(x) = sec^{-1} x$ Restrictions: Range is bounded Odd/Even: Even General Form: $f(x) = a\, sec\, \big(b(x-h)\big) + k$
Cotangent	$f(x) = cot\, x$ $= \dfrac{1}{tan\, x}$		Domain: $(-\infty, \infty)$ except for $x = \pm n\pi$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = cot^{-1} x$ Restrictions: Asymptotes at x = $\pm n\pi$ Odd/Even: Odd General Form: $f(x) = a\, cot\, \big(b(x-h)\big) + k$

Function Name	Parent Function	Graph	Characteristics
Arcsine	$f(x) = sin^{-1} x$		Domain: $[-1, 1]$ Range: $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ or Quadrants I & IV Inverse Function: $f^{-1}(x) = sin\, x$ Restrictions: Range & Domain are bounded Odd/Even: Odd General Form: $f(x) = a\, sin^{-1}\big(b(x-h)\big) + k$
Arccosine	$f(x) = cos^{-1} x$		Domain: $[-1, 1]$ Range: $[0,\ \pi]$ or Quadrants I & II Inverse Function: $f^{-1}(x) = cos\, x$ Restrictions: Range & Domain are bounded Odd/Even: None General Form: $f(x) = a\ cos^{-1}\ \big(b(x-h)\big) + k$
Arctangent	$f(x) = tan^{-1} x$		Domain: $(-\infty, \infty)$ Range: $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ or Quadrants I & IV Inverse Function: $f^{-1}(x) = tan\, x$ Restrictions: Range is bounded Odd/Even: Odd General Form: $f(x) = a\, tan^{-1}\ \big(b(x-h)\big) + k$
Arccosecant	$f(x) = csc^{-1} x$		Domain: $(-\infty, -1] \cup [1, \infty)$ Range: $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$ or Quadrants I & IV Inverse Function: $f^{-1}(x) = csc\, x$ Restrictions: Range & Domain are bounded Odd/Even: Odd General Form: $f(x) = a\ csc^{-1}\big(b(x-h)\big) + k$
Arcsecant	$f(x) = sec^{-1} x$		Domain: $(-\infty, -1] \cup [1, \infty)$ Range: $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$ or Quadrants I & II Inverse Function: $f^{-1}(x) = sec\, x$ Restrictions: Range & Domain are bounded Odd/Even: Neither General Form: $f(x) = a\, sec^{-1}\big(b(x-h)\big) + k$
Arccotangent	$f(x) = cot^{-1} x$		Domain: $(-\infty, \infty)$ Range: $(0, \pi)$ or Quadrants I & II Inverse Function: $f^{-1}(x) = cot\, x$ Restrictions: Range is bounded Odd/Even: Neither General Form: $f(x) = a\, cot^{-1}\ \big(b(x-h)\big) + k$

Function Name	Parent Function	Graph	Characteristics
Hyperbolics			
Hyperbolic Sine	$f(x) = \sinh x$ $= \dfrac{e^x - e^{-x}}{2}$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = sinh^{-1}\, x$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a\ sinh\ \big(b(x-h)\big) + k$
Hyperbolic Cosine	$f(x) = cosh\, x$ $= \dfrac{e^x + e^{-x}}{2}$		Domain: $(-\infty, \infty)$ Range: $[1, \infty)$ Inverse Function: $f^{-1}(x) = cosh^{-1}\, x$ Restrictions: None Odd/Even: Even General Form: $f(x) = a\ \ cosh\ \big(b(x-h)\big) + k$
Hyperbolic Tangent	$f(x) = tanh\, x$ $= \dfrac{e^{2x} - 1}{e^{2x} + 1}$		Domain: $(-\infty, \infty)$ Range: $(-1, 1)$ Inverse Function: $f^{-1}(x) = tanh^{-1}\, x$ Restrictions: Asymptotes at $y = \pm 1$ Odd/Even: Odd General Form: $f(x) = a\ tanh\ \big(b(x-h)\big) + k$
Hyperbolic Cosecant	$f(x) = csch\, x$ $= \dfrac{1}{sinh\, x}$		Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0] \cup [0, \infty)$ Inverse Function: $f^{-1}(x) = csch^{-1}\, x$ Restrictions: Asymptotes at $x = 0, y = 0$ Odd/Even: Odd General Form: $f(x) = a\ \ csch\ \big(b(x-h)\big) + k$
Hyperbolic Secant	$f(x) = \text{sech}\, x$ $= \dfrac{1}{cosh\, x}$		Domain: $(-\infty, \infty)$ Range: $(0, 1]$ Inverse Function: $f^{-1}(x) = sech^{-1}\, x$ Restrictions: Asymptote at $y = 0$ Odd/Even: Even General Form: $f(x) = a\ sech\ \big(b(x-h)\big) + k$
Hyperbolic Cotangent	$f(x) = coth\, x$ $= \dfrac{1}{tanh\, x}$		Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 1) \cup (1, \infty)$ Inverse Function: $f^{-1}(x) = coth^{-1}\, x$ Restrictions: Asymptotes at $x = 0, y = \pm 1$ Odd/Even: Odd General Form: $f(x) = a\ coth\ \big(b(x-h)\big) + k$

Function Name	Parent Function	Graph	Characteristics
Hyperbolic Arcsine	$f(x) = sinh^{-1} x$ $= ln(x + \sqrt{x^2 + 1})$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = sinh\, x$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a\, sinh^{-1}\big(b(x - h)\big) + k$
Hyperbolic Arccosine	$f(x) = cosh^{-1} x$ $= ln(x + \sqrt{x^2 - 1})$		Domain: $[1, \infty)$ Range: $[0, \infty)$ Inverse Function: $f^{-1}(x) = cosh\, x$ Restrictions: $y \geq 0$ Odd/Even: Neither General Form: $f(x) = a\ cosh^{-1}\ \big(b(x - h)\big) + k$
Hyperbolic Arctangent	$f(x) = tanh^{-1} x$ $= \frac{1}{2} ln\left(\frac{1 + x}{1 - x}\right)$		Domain: $(-1, 1)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = tanh\, x$ Restrictions: Asymptotes at $x = \pm 1$ Odd/Even: Odd General Form: $f(x) = a\, tanh^{-1}\ \big(b(x - h)\big) + k$
Hyperbolic Arccosecant	$f(x) = csch^{-1} x$ $= ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1}\right)$		Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0] \cup [0, \infty)$ Inverse Function: $f^{-1}(x) = csch\, x$ Restrictions: Asymptotes at $x = 0, y = 0$ Odd/Even: Odd General Form: $f(x) = a\ csch^{-1}\ \big(b(x - h)\big) + k$
Hyperbolic Arcsecant	$f(x) = sech^{-1} x$ $= ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right)$		Domain: $(0, 1]$ Range: $[0, \infty)$ Inverse Function: $f^{-1}(x) = sech\, x$ Restrictions: Odd/Even: Neither General Form: $f(x) = a\, sech^{-1}\ \big(b(x - h)\big) + k$
Hyperbolic Arccotangent	$f(x) = coth^{-1} x$ $= \frac{1}{2} ln\left(\frac{x + 1}{x - 1}\right)$		Domain: $[-\infty, -1) \cup (1, \infty]$ Range: $(-\infty, 0) \cup (0, \infty)$ Inverse Function: $f^{-1}(x) = coth\, x$ Restrictions: Asymptotes at $x = 0, y = \pm 1$ Odd/Even: Odd General Form: $f(x) = a\, coth^{-1}\ \big(b(x - h)\big) + k$

Graphing Tips

All Functions

The Seven Function "Levers"	*y = a f (b (x - h)) + k*	Graphing Tips
1) Move up/down ↕	k (Vertical translation)	"+" Moves it up
2) Move left/right ↔	h (Horizontal translation)	"+" Moves it right
3) Stretch up/down ↕	a (Vertical dilation)	Larger stretches it taller or makes it grow faster
4) Stretch left/right ↔	b (Horizontal dilation)	Larger stretches it out wider
5) Flip about x-axis ↻	a → −a	$f(x) \rightarrow -f(x)$ If $f(x) = -f(-x)$ then "odd" function
6) Flip about y-axis ↺	b → −b	$f(x) \rightarrow f(-x)$ If $f(x) = f(-x)$ then "even" function
7) Rotate CW/CCW ↺	$\cot 2\theta = \frac{A - C}{B}$	"+" θ rotates CCW For conic sections, where: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Trigonometric Functions

f(x) = a <trig> (b (x - h)) + k

The Six Trig "Levers"	*y = a sin (b (x - h)) + k*	Graphing Tips	Notes
1) Move up/down ↕	k (Vertical translation)	$k = \frac{(\max + \min)}{2}$	If $k = f(x)$ then x-axis is replaced by $f(x)$-axis
2) Move left/right ↔	h (Phase shift)	'+' shifts right	$sin\,(x) = cos\,(x - \pi/2)$
3) Stretch up/down ↕	a (Amplitude)	$a = \frac{(\max - \min)}{2}$	a is NOT peak-to-peak on y-axis
4) Stretch left/right ↔	b (Frequency • 2π)	$T = \frac{2\pi}{\lvert b \rvert} = \frac{1}{f}$	T = peak-to-peak on θ-axis $T = \frac{\pi}{\lvert b \rvert}$ for $tan\,(bx)$
5) Flip about y-axis ↺	b → −b	$f(x) \rightarrow f(-x)$	Even Function: $cos\,(x) = cos\,(-x)$
6) Flip about x-axis ↻	a → −a	$f(x) \rightarrow -f(-x)$	Odd Function: $sin\,(x) = -sin\,(-x)$