**Harold’s Probability**

**Cheat Sheet**

22 October 2022

**Probability**

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| **Rule** | **Formula** | **Definition** |
| **Notation** | $∩$ = “and”, Intersection, or “$∧$”$∪$ = “or”, Union, or “$∨$”$\overbar{˽}= $“not”, negation, or “$¬$” | “and” implies multiplication.“or” implies addition.“not” implies negation. |
| **Independent** | If $P\left(B\right)=P(A)$ | The occurrence of one event does not affect the probability of the other, or vice versa. |
| **Dependent** | If $P\left(A∩B\right)\ne ∅$ | The occurrence of one event affects the probability of the other event. |
| **Disjoint**(“mutually exclusive”) | If $P\left(A∩B\right)=∅$Then $P\left(A∪B\right)=P\left(A\right)+P\left(B\right)$ | The events can never occur together. |
| **Probability**(“likelihood”) | $$0\leq P\left(E\right)\leq 1$$$$P\left(E\right)=\frac{\# Events (E)}{Sample Space \left(S\right)} = \frac{\# of Favorable Outcomes}{Total \# of Possible Outcomes}$$ |
| **Addition Rule** (“or”) | $$P\left(A∪B\right)=P\left(A\right)+P\left(B\right)-P(A∩B)$$ | http://upload.wikimedia.org/wikipedia/commons/thumb/7/7b/Venn_A_intersect_B_alt.svg/235px-Venn_A_intersect_B_alt.svg.pngS |
| **Multiplication Rule**(“and”) | if independent or disjoint:$$P\left(A∩B\right)=P\left(A\right) P(B)$$$$P\left(A∩B∩C\right)=P\left(A\right) P\left(B\right) P(C)$$if dependent:$$P\left(A∩B\right)=P\left(A\right) P\left(A\right)$$$$P\left(A∩B\right)=P\left(B\right) P(A|B)$$$$P\left(A∩B\right)=P\left(A\right)-P\left(A∩\overline{B}\right)$$ |
| **Complement Rule / Subtraction Rule**(“not”) | $$P\left(S\right)=P\left(A∪\overline{A}\right)=$$$$P\left(A\right)+P\left(\overline{A}\right)=1$$$$P(A)=1-P\left(\overline{A}\right)$$$$P(\overline{A})=1-P\left(A\right)$$$$P\left(B\right)+P\left(B\right)=1$$ | The complement of event A (denoted $\overline{A} or A^{c})$ means “**not A**”; it consists of all simple outcomes that are not in A. |
| **Conditional Probability**(“given that”) | $$P\left(B\right)=\frac{P(A∩B)}{P(B)}$$if independent or disjoint:$$P\left(B\right)=P(A)$$$$P\left(A\right)=P(B)$$ | Means the probability of event A given that event B occurred. Is a rephrasing of the Multiplication Rule. P(A|B) is the proportion of elements in B that are ALSO in A. |
| **Total Probability Rule** | $$P\left(A\right)=P(A∩B\_{1})+…+P(A∩B\_{n})$$$$=P\left(B\_{1}\right) P\left(B\_{1}\right)+…+P\left(B\_{n}\right) P\left(B\_{n}\right)$$$$P\left(A\right)=P(A∩B)+P(A∩\overbar{B})$$$$=P\left(B\right) P\left(B\right)+P\left(\overbar{B}\right) P\left(\overbar{B}\right)$$ | To find the probability of event A, partition the sample space into several disjoint events. A must occur along with one and only one of the disjoint events. |
| **Bayes’ Theorem** | $$P\left(B\right)=\frac{P\left(A∩B\right)}{P\left(B\right)}=\frac{P\left(A\right) P\left(A\right)}{P\left(B\right)}$$$$=\frac{ P\left(A\right) P\left(A\right)}{P\left(A\right) P\left(A\right)+P\left(\overbar{A}\right) P\left(\overbar{A}\right)}$$ | Allows P(A|B) to be calculated from P(B|A).Meaning it allows us to reverse the order of a conditional probability statement, and is the only generally valid method! |
| **De Morgan’s Law** | $$\overbar{P\left(A∪B\right)}≡\overbar{P(A)}∩\overbar{P(B)}$$$$\overbar{P\left(A∩B\right)}≡\overbar{P(A)}∪\overbar{P(B)}$$ | Uses negation to convert an “or” to an “and”.Uses negation to convert an “and” to an “or”. |

**Discrete Distributions**

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| **Distribution** | **Formula** |
| **Probability Distribution** | $$\sum\_{s\in S}^{}p(s)=1$$ |
| **Factorial** | $$n!=n ⦁ \left(n-1\right) ⦁ \left(n-2\right) ⦁…⦁ 3 ⦁ 2 ⦁ 1$$ |
| **Permutation** | $$P(n,r)=\_{r}=\frac{n!}{\left(n-r\right)!}$$ |
| **Combination** | $$C(n,r)=\_{r}=\left(\genfrac{}{}{0pt}{}{n}{r}\right)=\frac{n!}{r!\left(n-r\right)!}$$ |
| **Uniform Discrete Distribution** | $$P\left(X=x\right)=\frac{1}{b-a+1}$$$$P\left(S=s\right)=\frac{1}{\left|S\right|} per outcome$$$$P\left(S=E\right)=\frac{\left|E\right|}{\left|S\right|} per event $$ |
| **Binomial Distribution** | $$P\left(X=k\right)=\left(\genfrac{}{}{0pt}{}{n}{k}\right)p^{k}\left(1-p\right)^{n-k}$$ |
| **Geometric Distribution** | $$P\left(X\leq x\right)=q^{x-1}p=(1-p)^{x-1}p$$$$P\left(X>x\right)=q^{x}=(1-p)^{x}$$ |
| **Poisson Distribution** | $$P\left(X=x\right)=\frac{λ^{x}e^{-λ}}{x!}, x=0,1,2,3,4,…$$ |
| **Bernoulli Distribution** | $$P\left(k;p\right)=p^{k}(1-p)^{1-k} for k \in \{0, 1\}$$ |
| **Trinomial Distribution** | $$P(X=x,Y=y)=\frac{n!}{x!y!(n-x-y)!}p\_{1}^{x}p\_{2}^{y}(1-p\_{1}-p\_{2})^{n-x-y}$$ |
| **Hypergeometric Distribution** | $$P(x|N, m, n)=\frac{\left(\left(\genfrac{}{}{0pt}{}{m}{x}\right)\left(\genfrac{}{}{0pt}{}{N-m}{n-x}\right)\right)}{\left(\genfrac{}{}{0pt}{}{N}{n}\right)}$$ |
| **Negative Binomial Distribution** | $$P(X=r)= \_{n+r-1}C\_{r-1} p^{r} q^{n}$$ |

**Venn Diagrams**



**B**

**Sources**:

* [SNHU MAT 229](https://www.snhu.edu/admission/academic-catalogs/coce-catalog#/courses/4188IbUYl) - Mathematical Proof and Problem Solving, [How To Prove It - A Structured Approach](https://www.amazon.com/How-Prove-Structured-Daniel-Velleman/dp/1108439535/ref%3Dsr_1_fkmr0_2?crid=3DLEIZI1MQFFK&keywords=How+To+Prove+It+-+A+Structured+Approach+3rd+Edition+-+Daniel+J.+Vellman&qid=1666431460&qu=eyJxc2MiOiIwLjgxIiwicXNhIjoiMC4wMCIsInFzcCI6IjAuMDAifQ%3D%3D&sprefix=how+to+prove+it+-+a+structured+approach+3rd+edition+-+daniel+j.+vellman%2Caps%2C131&sr=8-2-fkmr0&ufe=app_do%3Aamzn1.fos.18ed3cb5-28d5-4975-8bc7-93deae8f9840), 3rd Edition - Daniel J. Vellman, Cambridge University Press, 2019.
* [SNHU MAT 230](https://www.snhu.edu/admission/academic-catalogs/coce-catalog#/courses/4kVhSZLtg) - Discrete Mathematics, zyBooks.



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