**Harold’s Vectors Cheat Sheet**

5 December 2022

**Definitions**

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| **Term** | **Formula** | **Example** |
| **Vector Notation** | $$A, a$$ | Bold letter |
| $$\vec{a}, \vec{a}$$ | Arrow on top |
| **Component Notation** | $$a=\vec{a}=a\_{x}\hat{i}+a\_{y}\hat{j}+a\_{z}\hat{k}$$ | $$a=3\hat{i} +4\hat{j}+5\hat{k}$$ |
| $$a=\left〈a\_{x}, a\_{y},a\_{z}\right〉$$ | $$a=\left〈3, 4, 5\right〉$$ |
| $$a=r∠θ$$ | $a=5 ∠ 53.13°$(2D) |
|  | What are velocity components? (article) | Khan Academy**2D** | **3D** |
| **Vectors Used in Examples** | $$a=a\_{x}\hat{i} +a\_{y}\hat{j} +a\_{z}\hat{k}$$$$b=b\_{x}\hat{i} +b\_{y}\hat{j} +b\_{z}\hat{k}$$**2D**: set $a\_{z}=b\_{z}=0$ | $$a=3\hat{i} +4\hat{j}+5\hat{k}$$$$b=6\hat{i} -7\hat{j}-8\hat{k}$$ |
| **Magnitude** | $$\left‖a\right‖=\sqrt{a\_{x}^{2}+a\_{y}^{2}+a\_{z}^{2}}$$ | $$\left‖a\right‖=\sqrt{3^{2}+4^{2}+5^{2}}=\sqrt{50}=5\sqrt{2}$$ |
| Can also use $\left|a\right|$.Length of vector, but with no direction (scalar).Similar to a hypotenuse.Think multi-dimensional Pythagorean Theorem. | A vector |
| **Direction** | Divided into dimensional components. | A scalar with a direction is a vector.Example: speed vs. velocity |
| $$\tan(θ)=\frac{a\_{y}}{a\_{x}}$$ | $$θ=tan^{-1}\left(\frac{a\_{y}}{a\_{x}}\right)=tan^{-1}\left(\frac{4}{3}\right)≅53.13°$$ |
| **Unit Vector**(Basis Vector) | $$\hat{i}=x-axis=\left〈1, 0, 0\right〉$$$$\hat{j}=y-axis=\left〈0, 1, 0\right〉$$$$\hat{k}=z-axis=\left〈0, 0, 1\right〉$$ | Circumflex or “hat” on top.Indicates direction only.Always has a magnitude of one (1 or unit). |
| $$\vec{u}=\frac{a}{\left‖a\right‖}$$ | $$\vec{u}=\frac{a\_{x}\hat{i} +a\_{y}\hat{j} +a\_{z}\hat{k}}{\sqrt{a\_{x}^{2}+a\_{y}^{2}+a\_{z}^{2}}}$$ |
| **Scalar** | *k, m* | A number with no direction or units. |
| **Orthogonal** | A change in one dimension does not change in any of the values in the other dimensions. | 2D: right angle**Rectangular Coordinates:** The x-axis, y-axis, and z-axis are orthogonal to each other.**Polar Coordinates:** The angle is orthogonal to the line segment length |
| if $a•b=0$ | Two vectors are orthogonal if their dot product is zero. |
|  | See the source image | See the source image |
| **Parallel** | If $a=kb$ | Two vectors are parallel if they have the same direction. |
|  | Collinear of in opposite directions | See the source image |
| **Vector vs. Matrix** | vector = $1 x n$ or $n x 1$ matrix | A matrix with only one (1) row or column. |
|  | See the source image |

**Vector Operations**

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| **Operation** | **Formula** | **Example** |
| **Addition** | $$a+b= \left(a\_{x}+b\_{x}\right)\hat{i}+\left(a\_{y}+b\_{y}\right)\hat{j}+\left(a\_{z}+b\_{z}\right)\hat{k}$$ | $$a+b=\left(3+6\right)\hat{i}+\left(4-7\right)\hat{j}+\left(5-8\right)\hat{k}$$$$=9\hat{i}-3\hat{j}-3\hat{k}$$ |
| $$a+b=b+a$$ | Commutative |
| $$\left(a+b\right)+c=a+\left(b+c\right)$$ | Associative |
| $$(k+m)a=ka+ma$$ | Distributive |
| $$k\left(a+b\right)=ka+kb$$ |
|  |  | The opposite vector |
| **Subtraction** | $$a-b=a+\left(-b\right)$$ | Change the direction of $\vec{b}$ then add. |
|  | Parallel Two Vectors |
| **Scalar Multiplication** | $$ka=ka\_{x}\hat{i} +ka\_{y}\hat{j} +ka\_{z}\hat{k}$$ | $$3∙a=3∙3\hat{i} +3∙4\hat{j} +3∙5\hat{k}$$$$=9\hat{i} +12\hat{j} +15\hat{k}$$ |
| Changes the magnitude only. |
|  | $$k\left(a+b\right)=ka+kb$$ | Scalar multiplication - Wikipedia |
| **Dot Product**(Scalar Product) | $$a•b= a\_{x}b\_{x}+a\_{y}b\_{y}+a\_{z}b\_{z}$$ | $$a•b= (3)(6)+(4)(-7)+(5)(-8)=-50$$ |
| $$a•b=\left‖a\right‖ \left‖b\right‖\cos(θ)$$ | $$\cos(θ)=\frac{a•b}{\left‖a\right‖ \left‖b\right‖}$$ |
| $$a•b=b•a$$ | Commutative |
| $$a•\left(b+c\right)=a•b+a•c$$ | Distributive |
| $$k(a•b)=ka•b=a•kb$$ | Scalar Multiplication |
| $$0•u=0$$ | Zero Vector Dot Product |
| $$u•u=\left‖u\right‖^{2}$$ | Dot Product and Vector Magnitude Relationship |
| Is always a scalar. |
|  |  | 8: Dot product as projection onto a unit vector | Download Scientific  Diagram |
| **Cross Product**(Vector Product) | $$a⨯b=\left|\begin{matrix}i&j&k\\a\_{x}&a\_{y}&a\_{z}\\b\_{x}&b\_{y}&b\_{z}\end{matrix}\right|=\left|\begin{matrix}a\_{y}&a\_{z}\\b\_{y}&b\_{z}\end{matrix}\right|i-\left|\begin{matrix}a\_{x}&a\_{z}\\b\_{x}&b\_{z}\end{matrix}\right|j+\left|\begin{matrix}a\_{x}&a\_{y}\\b\_{x}&b\_{y}\end{matrix}\right|k$$$$=\left(a\_{y}b\_{z}-b\_{y}a\_{z}\right) i-\left(a\_{x}b\_{z}-b\_{x}a\_{z}\right) j+\left(a\_{x}b\_{y}-b\_{x}a\_{y}\right) k$$ |
| $$\left‖a⨯b\right‖=\left‖a\right‖ \left‖b\right‖\sin(θ)$$ | $$\sin(θ)=\frac{\left‖a⨯b\right‖}{\left‖a\right‖ \left‖b\right‖}$$ |
| $$a⨯b=-b⨯a$$ | Anti-Commutative |
| $$\left(a⨯b\right)⨯c\ne a⨯\left(b⨯c\right)$$ | Not Commutative |
| $$\left(a⨯b\right)⨯c=(a•c)b-(a•b) c$$ |
| $$a⨯(b⨯c)\ne (a⨯b)⨯c$$ | Not Associative |
| $$a•(b⨯c)=(a⨯b)•c$$ |
| $$a⨯\left(b+c\right)=a⨯b+a⨯c$$ | Distributive |
| $$\left(a+b\right)⨯c=a⨯c+b⨯c$$ |
| $$k(a⨯b)=ka⨯b=a⨯kb$$ | Scalar Multiplication |
| $$\left(ka\right)⨯b=k\left(a⨯b\right)=a⨯\left(kb\right)$$ |
| Is always a vector orthogonal to the other two vectors. |
|  |  | Cross products (article) | Khan Academy |
| **Scalar Triple Product** | $$\left(a⨯b\right)•c=a•\left(b⨯c\right)=b•\left(c⨯a\right)=c•\left(a⨯b\right)=\left|\begin{matrix}a\_{x}&a\_{y}&a\_{z}\\b\_{x}&b\_{y}&b\_{z}\\c\_{x}&c\_{y}&c\_{z}\end{matrix}\right|$$ |
| **Vector Triple Product** | $$a⨯\left(b⨯c\right)=\left(a•c\right) b-\left(a•b\right) c$$ |

**Vector Applications**

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| **Application** | **Formula** | **Example** |
| **Projection** | $$proj\_{a}b=\left(\frac{a∙b}{a∙a}\right)a$$ | The dot product - Math Insight |
| **Right Hand Rule** | The cross product produces a vector orthogonal to the other two vectors. | Use the right hand rule to determine direction of the cross product vector. |
|  | Right hand rule and the cross product |
| **Area**(Parallelagram) | $$A=\left‖a⨯b\right‖$$ | Vector Product - Cross Product |
| **Volume**(Parallelepiped) | $$V=\left‖\left(a⨯b\right)•c\right‖$$ | 2.4 Products of Vectors | University Physics Volume 1 |
| **Torque** | $$τ=r⨯F$$ | $$\left‖τ\right‖=r F\sin(θ)$$ |
| **Coplanar** | Three vectors are coplanar if $$\left(a⨯b\right)•c=\left|\begin{matrix}a\_{x}&a\_{y}&a\_{z}\\b\_{x}&b\_{y}&b\_{z}\\c\_{x}&c\_{y}&c\_{z}\end{matrix}\right|=0$$ | All three vectors are in the same plane. |