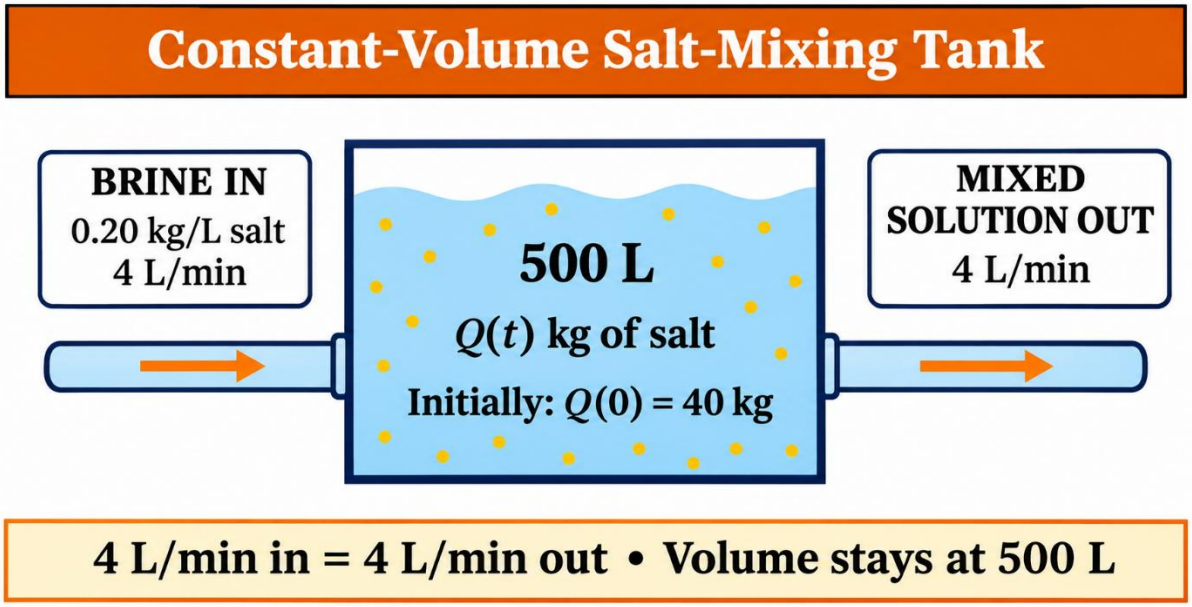
Harold's Brine Mixture
Example Sheet
10 November 2026



A tank initially contains 500 L of solution with 40 kg of dissolved salt. Brine containing 0.20 kg/L of salt enters at 4 L/min. The thoroughly mixed solution drains at 4 L/min, so the volume does not change.

a. **Set up and solve a differential equation for the amount of salt in the tank, $Q(t)$.**

b. **Find the amount of salt in the tank after 1½ hours.**

Let $Q(t)$ be the amount of salt in kilograms and let t be time in minutes.
The volume stays constant at 500 L.

1. Set up the differential equation

Salt entering per minute:

$$(0.20 \text{ kg/L})(4 \text{ L/min}) = 0.8 \text{ kg/min}.$$

Salt leaving per minute:

The salt concentration in the tank is $\frac{Q(t)}{V} = \frac{Q(t)}{500}$ kg/L, so salt leaves at

$$4\left(\frac{Q(t)}{500}\right) = 0.008\,Q(t) \text{ kg/min}.$$

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Using rate of change = rate in − rate out:

$$\boxed{\frac{dQ}{dt} = 0.8 - 0.008\,Q, \qquad Q(0) = 40.}$$

2. Solve the differential equation

Since $0.8 = 0.008(100)$, rewrite and separate variables:

$$\frac{dQ}{dt} = 0.008\,(100 - Q)$$

$$\frac{dQ}{100 - Q} = 0.008\,dt.$$

Integrate both sides:

$$\int \frac{dQ}{100 - Q} = \int 0.008\,dt$$

$$-\ln|100 - Q| = 0.008\,t + C$$

Exponentiate to get:

$$|100 - Q| = Ae^{-0.008t}$$

The solution stays below 100 kg, so $100 - Q > 0$ and the absolute-value bars can be removed.

$$100 - Q = Ae^{-0.008t}, \qquad A > 0.$$

Solving for Q gives:

$$Q(t) = 100 - Ae^{-0.008t}$$

Applying the initial condition $Q(0) = 40$ gives:

$$40 = 100 - A(1)$$
$$A = 60$$

Therefore,

$$\boxed{Q(t) = 100 - 60e^{-0.008t}\ \text{kg}}$$

3. Find the amount of salt after 1½ hours

Convert hours to minutes.

$$1.5\ \cancel{\text{hours}} \cdot \left(\frac{60\ \text{min}}{\cancel{\text{hour}}}\right) = 90\ \text{min}$$

$$Q(90) = 100 - 60e^{-0.008(90)} = 100 - 60e^{-0.72}$$

$$\boxed{Q(90) \approx 70.795\ \text{kg}}$$

The model approaches 100 kg of salt over time.

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