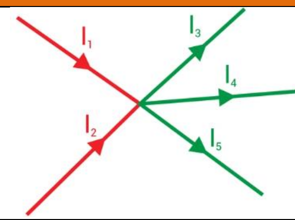
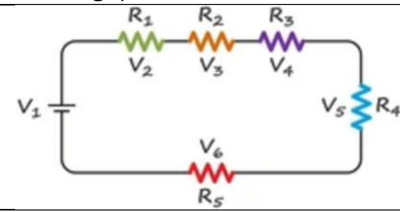
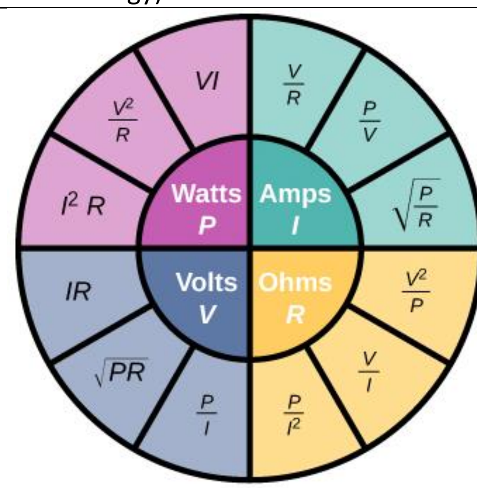
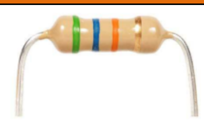
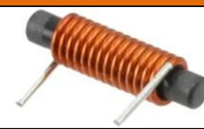
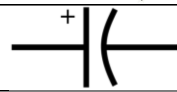
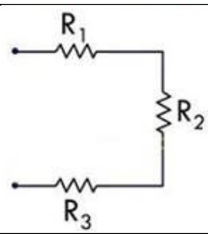
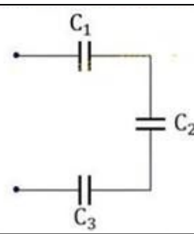
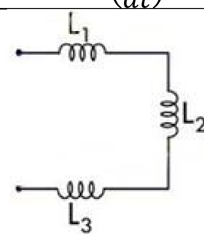
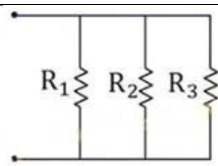
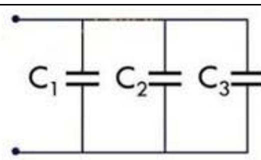
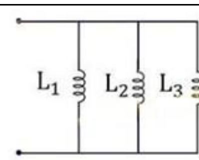
Harold's AC Circuits
Cheat Sheet
2 December 2024

Circuit Laws

<table>
<tr><th>Circuit Law</th><th colspan="2">Formula</th></tr>
<tr><td rowspan="2">Kirchhoff's Current Law (KCL)</td><td>$$\sum_{i=1}^{n} I_i = 0$$ $$I_{in1} + I_{in2} = I_{out1} + I_{out2} + I_{out3}$$</td><td>
</td></tr>
<tr><td colspan="2">The total current flowing into a node or junction must equal the total current flowing out of the node or junction.
(conservation of charge)</td></tr>
<tr><td rowspan="2">Kirchhoff's Voltage Law (KVL)</td><td>$$\sum_{i=1}^{n} V_i = 0$$ $$V_1 = V_2 + V_3 + V_4 + V_5 + V_6$$</td><td>
</td></tr>
<tr><td colspan="2">The sum of all voltages around a circuit loop is equal to zero.
(conservation of energy)</td></tr>
<tr><td>Ohm's Law</td><td>$$V = IR \ \ (DC\ circuit)$$ $$V = IZ_{Eq} \ \ (AC\ circuit)$$</td><td>
</td></tr>
<tr><td>Power</td><td>$$P = VI$$ $$P = I^2 R$$ $$P = \frac{V^2}{R}$$ $$P = \frac{W}{t}$$</td><td>An across (V) times a through (I) variable.
$$Watt\ (W) = \frac{Volt\ (V)}{Ampere\ (A)}$$</td></tr>
<tr><td>Electrical Energy</td><td>$$E = QV$$ $$E = VIt$$ $$P = I^2 Rt$$ $$E = Pt$$</td><td>$$Joule\ (J) = \frac{Watt\ (W)}{second\ (s)}$$</td></tr>
</table>

Components in Series/Parallel

Element	Resistor	Capacitor	Inductor
Component			
Symbol			
Denoted by	R	C	L
Units	Ω (Ohm)	F (Farad)	H (Henry)
Equation	$R = \frac{V_R}{I}$	$C = \frac{Q}{V_C}$	$L = \frac{V_L}{\left(\frac{di}{dt}\right)}$
Series			
	$R_T = R_1 + R_2 + R_3$	$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$	$L_T = L_1 + L_2 + L_3$
Parallel			
	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$	$C_T = C_1 + C_2 + C_3$	$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$

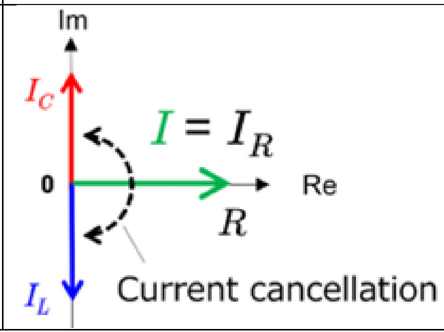
Impedance

Component	Units	Impedance (Z)	Phasor Notation	Complex Notation
Resistance (R)	Ω (Ohm)	$Z_R = R$	$R \angle 0°$	$R + 0j$
Inductance (L)	mH (Henry)	$Z_L = j\omega L$	$\lvert Z_L \rvert \angle 90°$	$0 + \lvert Z_L \rvert j$
Capacitance (C)	µF (Farad)	$Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$	$\lvert Z_C \rvert \angle -90°$	$0 - \lvert Z_C \rvert j$
Impedance (Z)	Ω (Ohm)	$Z_{eq} = Z_R + Z_L + Z_C$	$\lvert Z_{eq} \rvert \angle \theta°$	$Z_{Re} + Z_{Im}j$
Alternating Voltage (V)	V (Volts)	$V = IZ_{eq}$	$\lvert V_{eq} \rvert \angle \theta°$	$V_{Re} + V_{Im}j$
Alternating Current (I)	A (Ampere)	$I_{eq} = \frac{V}{Z_{eq}}$	$\lvert I_{eq} \rvert \angle \theta°$	$I_{Re} + I_{Im}j$
NOTE: In circuits, j is used to denote $\sqrt{-1}$ instead of i, which is already used for current.				

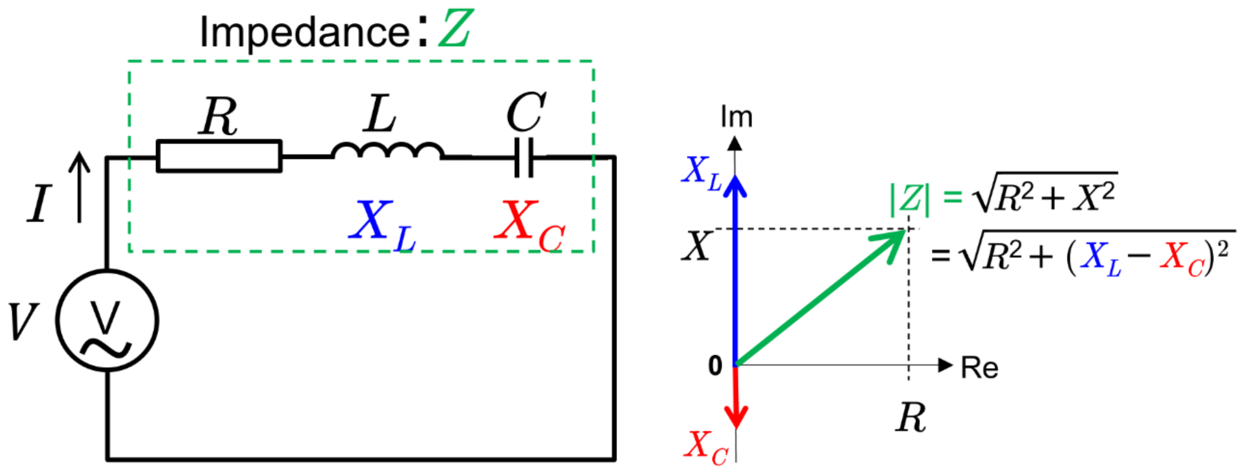
Phasor Math

Circuit Law	Formula	TI-84 Calculator
Phasor (Polar)	$\lvert Z\rvert \angle \theta° = \lvert Z\rvert e^{\theta j}$ $80 \angle -30° = 80e^{-30j}$	$Example: 80 \angle -30°$ [MODE] RADIAN [MODE] $re\text{^}(\theta i)$ [2nd] [QUIT] [8] [0] [2nd] [e^x] [-] [3] [0] [2nd] [π] [÷] [180] [2nd] [i] [ENTER]
Complex (Rectangular)	$Z_{Re} + Z_{Im}j$	$Example: 2 + 3i$ [MODE] DEGREE [MODE] a+bi [2nd] [QUIT] [2]+[3][2nd] [i] [ENTER]
Addition	$\lvert Z_1\rvert \angle \theta_1^{°} + \lvert Z_2\rvert \angle \theta_2^{°}$ 1. Convert from polar to rectangular form 2. Add real to real and imaginary to imaginary 3. Convert back from rectangular to polar form	
Division	$\frac{\lvert Z_1\rvert \angle \theta_1^{°}}{\lvert Z_2\rvert \angle \theta_2^{°}} = \left\lvert\frac{Z_1}{Z_2}\right\rvert \angle \left(\theta_1^{°} - \theta_2^{°}\right)$	
Rectangular → Polar	$Z = Z_{Re} + Z_{Im}j \rightarrow \lvert Z\rvert \angle \theta$ $\lvert Z\rvert = \sqrt{Z_{Re}^2 + Z_{Im}^2}$ $\theta = \tan^{-1}\left(\frac{Z_{Im}}{Z_{Re}}\right)$	$Example: 2 + 3i$ [MODE] DEGREE [MODE] $re\text{^}(\theta i)$ [2nd] [QUIT] [2] + [3] [2nd] [i] [ENTER] [ANS] = $3.61e^{56.31i}$ [2] + [3] [2nd] [i] [ENTER] [MATH][CPX][▶Polar] [ENTER]
Polar → Rectangular	$Z = \lvert Z\rvert \angle \theta \rightarrow Z_{Re} + Z_{Im}j$ $Z_{Re} = \lvert Z\rvert \cos(\theta)$ $Z_{Im} = \lvert Z\rvert\, sin(\theta)$	$Example: 80 \angle -30°$ [MODE] RADIAN [MODE] a+bi [2nd] [QUIT] 8] [0] [2nd] [e^x] [-] [3] [0] [2nd] [π] [÷] [180] [2nd] [i] [ENTER] [ANS] = 69.28 – 40i 8] [0] [2nd] [e^x] [-] [3] [0] [2nd] [π] [÷] [180] [2nd] [i] [ENTER] [MATH][CPX][▶Rect] [ENTER]

Resonance

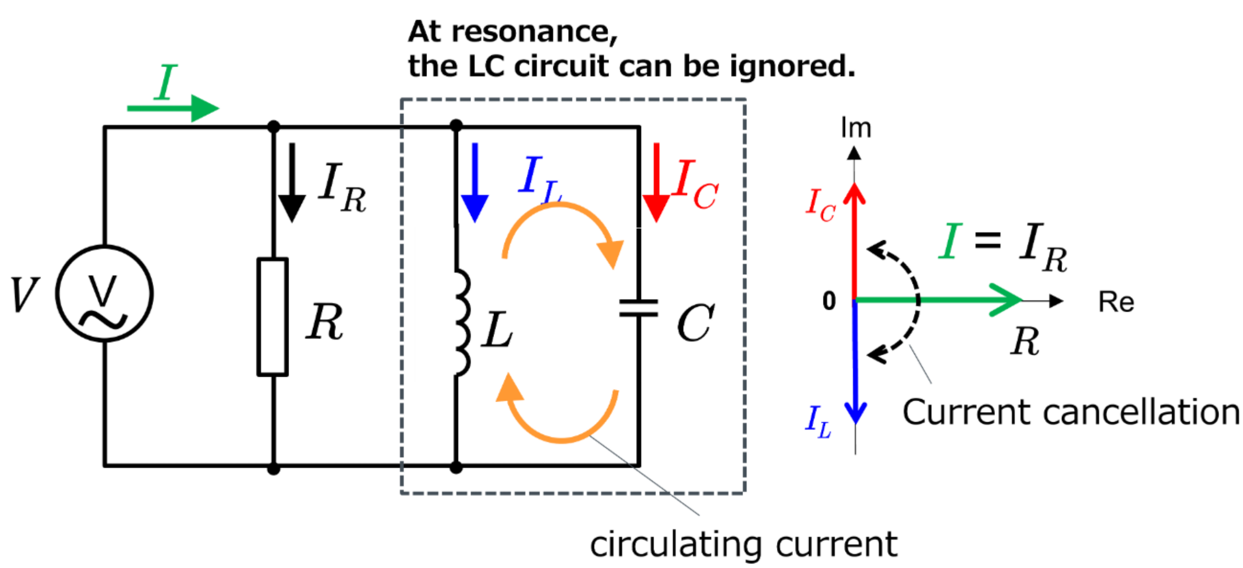
Term	Formula	TI-84 Calculator
Frequency (Hz)	$\omega = 2\pi f$ $\frac{\omega^\circ}{sec} = \omega\left(\frac{\pi}{180^\circ}\right)\frac{rad}{sec}$	[MODE] DEGREE [MODE] RADIAN
Resonance Frequency	$Z_L + Z_C = 0$ $\omega_0 = \frac{1}{\sqrt{LC}}\frac{rad}{s}$ $f_0 = \frac{1}{2\pi\sqrt{LC}}Hz$	

AC Circuit Example #1: Series RLC



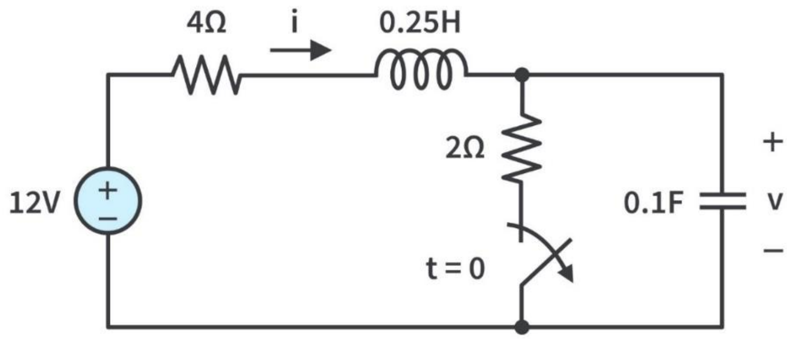
$$V = Z_{eq}I$$
$$V = (Z_R + Z_L + Z_C)I$$
$$V = \left(R + j\omega L + \frac{1}{j\omega C}\right)I = \left(R + j\omega L - j\frac{1}{\omega C}\right)I$$

AC Circuit Example #2: Parallel RLC



$$V = Z_{eq}I$$
$$V = \left(\frac{1}{\frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}}\right)I = \left(\frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C}\right)I$$

AC Circuit Example #3: Series (RL) and Parallel (RC)



1. Determine the impedance from series and parallel components using complex notation.

$$Z_{eq} = (Z_{R1} + Z_{\mathrm{L}}) + \left(\frac{1}{\frac{1}{Z_{R2}} + \frac{1}{Z_C}}\right)$$

$$Z_{eq} = (R_1 + j\omega L) + \left(\frac{1}{\frac{1}{R_2} + \frac{1}{1/_{j\omega C}}}\right)$$

2. Substitute values.

$$Z_{eq} = (4 + (0.25)(10^{-3})\omega j) + \left(\frac{1}{\frac{1}{2} - (0.1)(10^{-6})\omega j}\right)$$

$$Z_{eq} = (4 + 2.5 \bullet 10^{-4}\omega j) + \left(\frac{1}{0.5 - 10^{-7}\omega j}\right)$$

$$Z_{eq} = (4 + 2.5 \bullet 10^{-4}\omega j) + \left(\frac{0.5}{0.25 + 10^{-14}\omega^2} + \frac{10^{-7}\omega}{0.25 + 10^{-14}\omega^2}j\right)$$

$$Z_{eq} = \left(4 + \frac{0.5}{0.25 + 10^{-14}\omega^2}\right) + \left(2.5 \bullet 10^{-4}\omega + \frac{10^{-7}\omega}{0.25 + 10^{-14}\omega^2}\right)j$$

3. Convert from complex to polar notation.

$$Z_{eq} = Z_{Re} + Z_{Im}j$$

$$Z_{Re} = \left(4 + \frac{0.5}{0.25 + 10^{-14}\omega^2}\right)$$

$$Z_{Im} = \left(2.5 \bullet 10^{-4}\omega + \frac{10^{-7}\omega}{0.25 + 10^{-14}\omega^2}\right)$$

$$|Z_{eq}| = \sqrt{Z_{Re}^2 + Z_{Im}^2}$$

$$\theta = \tan^{-1}\left(\frac{Z_{Im}}{Z_{Re}}\right)$$

$$Z_{eq} = |Z_{eq}| \angle \theta$$

Sources:

- Electrical Technology (2020 Oct). Resistance, Capacitance & Inductance in Series-Parallel – Equation & Formulas. https://www.electricaltechnology.org/2020/10/resistance-inductance-capacitance-series-parallel-formulas.html
- LibreTexts Physics (2024). 9.6: Electrical Energy and Power. https://phys.libretexts.org/Bookshelves/University_Physics/University_Physics_(OpenStax)/University_Physics_II_-_Thermodynamics_Electricity_and_Magnetism_(OpenStax)/09%3A_Current_and_Resistance/9.06%3A_Electrical_Energy_and_Power
- Tech Web (2024). Resonant Circuits: Resonant Frequency and Q Factor. https://techweb.rohm.com/product/power-device/si/18332/