**Harold’s Algebra**

**Cheat Sheet**

22 September 2025

**Arithmetic Operations (+, –, ⦁, ÷)**

|  |  |  |
| --- | --- | --- |
| **Property** | **Equation** | **Tips** |
| **Commutative** |  | Reordering does not change the answer |
| **Associative** |  | Regrouping does not change the answer |
| **Distributive** |  | Extract common terms () |
| **Identity** |  | No change  0 plus anything = anything  1 times anything = anything |
| **Unity** |  | Used often  Assuming |
| **Sum (Addition)** |  | Need a common denominator |
| **Difference (Subtraction)** |
| **Product (Multiplication)** |  | Multiply numerator with numerator and denominator with denominator |
| **Quotient (Division)** |  | Invert and multiply |
| **Cross Multiplication** |  | Used often |
| **Variable Fraction** |  | Set , then  multiply numerator with numerator and denominator with denominator |
| **Variable Fraction** |  | Invert then multiply |
| **Distribute Denominator** |  | divides both sides of the numerator |
| **Swapping Signs** |  | Multiplying both the numerator and denominator by (-1) cancels the negatives |
| **Cancelling Terms** |  | Assuming |
| **Fraction over a Fraction** |  | Invert the denominator then multiply |
| **Fraction from Hell** |  |  |

**Exponents ()**

|  |  |  |
| --- | --- | --- |
| **Property** | **Equation** | **Example** |
| **Product Rule** |  |  |
| **Power Rule** |  |  |
| **Zero Power** |  | By definition assuming |
| **Negative Powers** |  |  |
| **Quotient Rule** |  |  |
| **Product Distribution** |  |  |
| **Quotient Distribution** |  |  |
| **Fractional Powers** |  |  |
| **Negative Powers** |  |  |
| **Domain** | The domain of is | Exponentials |

**Radicals ()**

|  |  |  |
| --- | --- | --- |
| **Property** | **Equation** | **Example** |
| **Fractional Powers** |  |  |
| **Power Rule**  **(Nested Roots)** |  |  |
| **Product Distribution** |  |  |
| **Quotient Distribution** |  |  |
| **Odd** | if is odd | Odd powers always keep their sign |
| **Even** | if is even | Even roots of even powers can be both positive and negative.  Substitute to check if each solution is valid. |
| **Domain** | The domain of is | Horiziontal Translation of Square Root Graphs - Definition - Expii |

**Logarithms ()**

|  |  |  |  |
| --- | --- | --- | --- |
| **Property** | **Equation** | | **Example** |
| **Definition** |  | | Pulls the exponent down to level |
| **Example** |  | | Logs make large numbers very small |
| **Special Logs** | Natural log | |  |
| = Logarithme Naturel (French) | | where e ≈ 2.718281828 … |
| Common log | |  |
| Computer science log | |  |
| Pre-1950 US math textbooks log | |  |
| **Rules** | **Common** | **Natural** |  |
| **Trivial Identities** |  |  |  |
|  |  |  |
| **Cancelling Exponentials** |  |  | Inverse function |
|  |  | Inverse function |
| **Exponential Rule** |  | | Convert to first |
| **Product Rule** |  | | Basis of slide rules |
| **Quotient Rule** |  | |  |
| **Change of Base** |  | | is the new base  TI-84: [MATH] [MATH] [A:logBASE(] |
| **Base Switch** |  | | Can use instead of TI-84 function |
| **Obscure Rules** |  | | Useful in probability theory |
| **Domain** | The domain of is | | Calculus I - Logarithm Functions |

**Factoring Formulas**

|  |  |  |  |
| --- | --- | --- | --- |
| **Description** | **Equation** | | **Notes** |
| **Quadratic** |  | |  |
| **Cubic** |  | | Binomial expansion (Pascal’s Triangle)  Notice one ‘‘ and two ‘’ for both |
| **Even Powers** |  | | The top equation is not often used since it requires imaginary numbers () |
| **Odd Powers** |  | | |
| **End-Terms Method** |  | Four possibilities to get middle-term:   1. Select one 3 2. Select one 5 | |
| **Box Method** |  |  | |
| **Visual Method** |  | | |

**Quadratic Formula (x2)**

|  |  |  |
| --- | --- | --- |
| **Description** | **Equation** | **Notes** |
| **Quadratic Formula Form** |  | Remember, and can be negative. |
| **Quadratic Formula** |  | * : two real unequal roots * : repeated real roots (multiplicity of two) * : two complex roots |
| **Completing the Square** | 1. Start with the quadratic formula form 2. Divide both sides by 3. Subtract the constant on the left to move it to the right side 4. Cut the middle term in half, square it, then add it to both sides 5. Use an identity to factor the left side 6. Take the square root of both sides 7. Solve for by subtracting the constant on the left to move it to the right | |
| **Vertex** | Vertex:  Finds the minimum / maximum of a parabola. | |
| Calculus method: Power Rule | |
|  | | |

**Cubic Formula (x3)**

|  |  |  |
| --- | --- | --- |
| **Description** | **Equation** | **Notes** |
| **Cubic Formula Form** |  | Coefficients must be integers |
| **Cubic Formula** | where  Find and using the quadratic formula. | |

**Inequalities ()**

|  |  |  |
| --- | --- | --- |
| **Property** | **Equation** | **Notes** |
| **Inequality Symbols** |  |  |
| **Sum** |  |  |
| **Difference** |  |  |
| **Product (Positive)** |  | Multiplying or dividing by a positive number does not change the inequality direction |
| **Quotient (Positive)** |  |
| **Product (Negative)** |  | Multiplying or dividing by a negative number changes the inequality direction  (e.g., ) |
| **Quotient (Negative)** |  |
| **Inequality Notation** | **Interval Notation** | **Number Line Graph** |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  | A black line with a red circle and black text  Description automatically generated |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

**Absolute Values ()**

|  |  |  |
| --- | --- | --- |
| **Property** | **Equation** | **Notes** |
| **Definition** |  | Simply strip away the minus sign |
| **Observations** |  | Are never negative |
|  | We care about the magnitude only |
| **Product Rule** |  |  |
| **Quotient Rule** |  |  |
| **Solving** |  | Positive Case:  Negative Case: |
| **Triangle Inequality** |  | Triangle inequality - Wikipedia |

**Distance Formula**

|  |  |
| --- | --- |
| **Property** | **Equation** |
| **Distance Formula** | Same as Pythagorean Theorem |
| **Graph** | Distance between two points - Formula and examples - Neurochispas |

**Complex Numbers ()**

|  |  |  |
| --- | --- | --- |
| **Property** | **Equation** | **Notes** |
| **Imaginary Number**  (Adjacent Number) |  |  |
| **Usage** |  | Assuming |
| **Complex Number** |  | Real + Imaginary |
| **Sum** |  | |
| **Difference** |  | |
| **Product** |  | |
| **Quotient** |  | |
| **Conjugate** |  |  |
| **Modulus** |  | Distance from origin to |
| **Squares** |  | |
| **Euler’s Identity** |  | |
| **Graph** | A graph of a function  Description automatically generated | |

**Conic Sections**

A different shapes of geometric shapes

Description automatically generated with medium confidence

**Line**

|  |  |  |
| --- | --- | --- |
| **Description** | **Equation** | **Notes** |
| **Slope** |  | Passes through points and |
| **Standard Form** |  | Where is positive |
| **Slope-Intercept Form** |  | is the -axis intercept  Passes through point |
| **Point-Slope Form** |  | Manipulation of the slope formula |
| **Intercept Form** |  | is the -intercept  is the -intercept |
| **Vertical Line** |  | Passes through point |
| **Horizontal Line** |  | Passes through point |
| **Domain** | The domain of  is |  |

**Circle**

|  |  |  |
| --- | --- | --- |
| **Description** | **-Axis Symmetry** | **-Axis Symmetry** |
| **Standard Form** |  | Same |
| **Radius** |  | Same |
| **Center** |  | Same |
| **Vertices** | None | Same |
| **Focus** |  | Same |
| **Graph** |  | Same |

**Parabola**

|  |  |  |
| --- | --- | --- |
| **Description** | **-Axis Symmetry** | **-Axis Symmetry** |
| **Standard Form** |  |  |
| **Vertex Form** |  |  |
| **Factored Form** |  | Roots |
| **Graphing Form** |  |  |
| **Vertex** |  |  |
| **Focus** |  |  |
| **Directrix** |  |  |
| **Direction** | Opens up if  Opens down if | Opens right if  Opens left if |
| **Graph** |  |  |

**Ellipse**

|  |  |  |
| --- | --- | --- |
| **Description** | **-Axis Symmetry** | **-Axis Symmetry** |
| **Standard Form** |  |  |
| **Center** |  |  |
| **Vertices** |  |  |
| **Co-Verticies** |  |  |
| **Foci** |  |  |
| **Focus Length** |  |  |
| **Graph** |  |  |
| *The sum of the distances from each focus to a point on the curve is constant.* | | |

**Hyperbola**

|  |  |  |
| --- | --- | --- |
| **Description** | **-Axis Symmetry** | **-Axis Symmetry** |
| **Standard Form** |  |  |
| **Center** |  |  |
| **Vertices** |  |  |
| **Foci** |  |  |
| **Focus Length** |  |  |
| **Asymptotes** |  |  |
| **Direction** | Opens left/right | Opens up/down |
| **Graph** |  |  |
| The difference between the distances from each focus to a point on the curve is constant. | | |

**Sources:**

* Brack, Tyne (2023, 24 January). Maneuvering the Middle, Factoring Polynomials with Special Cases. <https://www.maneuveringthemiddle.com/factoring-polynomials-with-special-cases/>
* Dawkins, Paul (2024). Paul’s Online Notes, Algebra Cheat Sheet. <https://tutorial.math.lamar.edu>
* Larson, Ron & Edwards, Bruce (2009). Algebra 1-Pager Reference, Calculus 9th Edition. <https://www.amazon.com/Bruce-Edwards-Larson-Calculus-Hardcover/dp/B004QICACS/ref=sr_1_1>
* Math Salamanders (2024). Inequalities on a Number Line. <https://www.math-salamanders.com/inequalities-on-a-number-line.html>
* TeachersPayTeachers.com (2019). Conic Section Graphs, All Things Algebra. <https://www.pinterest.com/pin/612911830514800210/>

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Description automatically generated with medium confidence