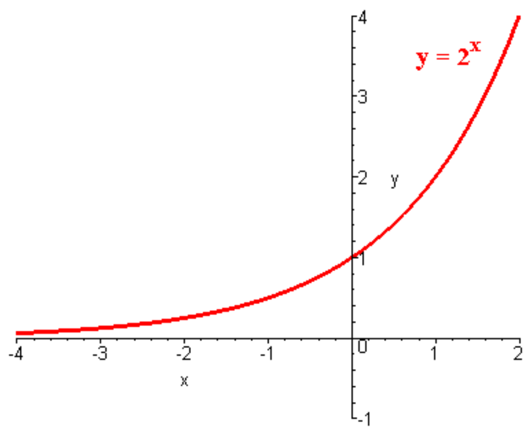
Harold's Algebra
Cheat Sheet
22 September 2025

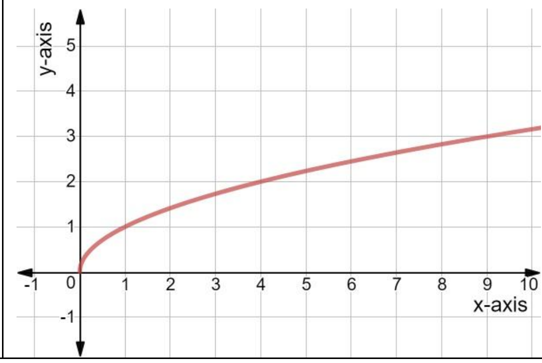
Arithmetic Operations (+, –, •, ÷)

Property	Equation	Tips
Commutative	$a + b = b + a$ $a \cdot b = b \cdot a$	Reordering does not change the answer
Associative	$(a + b) + c = a + (b + c)$ $(a \cdot b) \cdot c = a \cdot (b \cdot c)$	Regrouping does not change the answer
Distributive	$a(b \pm c) = ab \pm ac$	Extract common terms (a)
Identity	$a + 0 = a$ $a \cdot 1 = a$	No change 0 plus anything = anything 1 times anything = anything
Unity	$\left(\frac{anything}{anything}\right) = 1$	Used often Assuming $anything \neq 0$
Sum (Addition) **Difference (Subtraction)**	$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$	Need a common denominator $\left(\frac{a}{b}\right)\left(\frac{\boldsymbol{d}}{\boldsymbol{d}}\right) + \left(\frac{c}{d}\right)\left(\frac{\boldsymbol{b}}{\boldsymbol{b}}\right) = \frac{ad + bc}{bd}$
Product (Multiplication)	$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	Multiply numerator with numerator and denominator with denominator
Quotient (Division)	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$	Invert and multiply
Cross Multiplication	$If\ \frac{a}{b} = \frac{c}{d}\ then\ ad = bc$	Used often
Variable × Fraction	$a\left(\frac{b}{c}\right) = \frac{ab}{c}$	Set $a = \frac{a}{1}$, then multiply numerator with numerator and denominator with denominator
Variable ÷ Fraction	$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$	Invert $\left(\frac{b}{c}\right)$ then multiply
Distribute Denominator	$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$	c divides both sides of the numerator
Swapping Signs	$\frac{a - b}{c - d} = \frac{b - a}{d - c}$	Multiplying both the numerator and denominator by (-1) cancels the negatives
Cancelling Terms	$\frac{ab + ac}{a} = b + c$	Assuming $a \neq 0$
Fraction over a Fraction	$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$	Invert the denominator then multiply
Fraction from Hell	$\frac{\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}}{\frac{\left(\frac{e}{f}\right)}{\left(\frac{g}{h}\right)}} = \frac{adfg}{bceh}$	$\frac{\left(\frac{\left(\frac{N}{D}\right)}{\left(\frac{D}{N}\right)}\right)}{\left(\frac{\left(\frac{D}{N}\right)}{\left(\frac{N}{D}\right)}\right)} = \frac{NNNN}{DDDD}$

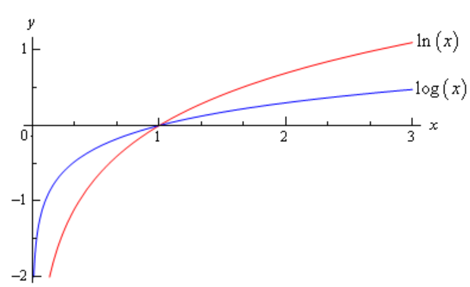
Exponents (a^b)

Property	Equation	Example
Product Rule	$a^n a^m = a^{n+m}$	$a^3 a^2 = (a \cdot a \cdot a)(a \cdot a) = a^{3+2} = a^5$
Power Rule	$(a^n)^m = a^{nm}$	$(a^3)^2 = (a \cdot a \cdot a)(a \cdot a \cdot a) = a^{3 \cdot 2} = a^6$
Zero Power	$a^0 = 1$	By definition assuming $a \neq 0$
Negative Powers	$a^{-n} = \frac{1}{a^n}$ $\frac{1}{a^{-n}} = a^n$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
Quotient Rule	$\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$	$\frac{2^4}{2^3} = \frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 2^{4-3} = 2^1 = 2$
Product Distribution	$(ab)^n = a^n b^n$	
Quotient Distribution	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{8}$
Fractional Powers	$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}}$	$2^{\frac{3}{4}} = \left(2^{\frac{1}{4}}\right)^3 = (2^3)^{\frac{1}{4}}$
Negative Powers	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$	$\left(\frac{1}{2}\right)^{-3} = \left(\frac{2}{1}\right)^3 = 8$
Domain	The domain of a^x is $(-\infty, \infty)$	

Radicals ($\sqrt[n]{a}$)

Property	Equation	Example
Fractional Powers	$\sqrt[n]{a} = a^{\frac{1}{n}}$	$\sqrt[3]{2} = 2^{\frac{1}{3}}$
Power Rule (Nested Roots)	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a}$	$\sqrt[4]{\sqrt[3]{2}} = \sqrt[3\cdot 4]{2} = \sqrt[12]{2}$
Product Distribution	$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$	$\sqrt[3]{14} = \sqrt[3]{2}\,\sqrt[3]{7}$
Quotient Distribution	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	$\sqrt[4]{\frac{2}{3}} = \frac{\sqrt[4]{2}}{\sqrt[4]{3}}$
Odd	$\sqrt[n]{a^n} = a$ if n is odd	Odd powers always keep their sign
Even	$\sqrt[n]{a^n} = \pm a$ if n is even	Even roots of even powers can be both positive and negative. Substitute to check if each solution is valid.
Domain	The domain of $\sqrt{x}$ is $x \geq 0$	

Logarithms ($\log_b x$)

Property	Equation		Example
Definition	$b^y = x \iff \log_b(x) = y$		Pulls the exponent down to level
Example	$5^3 = 125 \iff \log_5(125) = 3$		Logs make large numbers very small
Special Logs	Natural log ln = Logarithme Naturel (French)		$\ln(x) = \log_e(x)$ where e ≈ 2.718281828 …
	Common log		$\log(x) = \log_{10}(x)$
	Computer science log		$\log(x) = \log_2(x)$
	Pre-1950 US math textbooks log		$\log(x) = \log_e(x) = \ln(x)$
Rules	**Common**	**Natural**	
Trivial Identities	$\log_b(b) = 1$	$ln(e) = 1$	$b^1 = b$
	$\log_b(1) = 0$	$ln(1) = 0$	$b^0 = 1$
Cancelling Exponentials	$\log_b(b^x) = x$	$ln(e^x) = x$	Inverse function $f\left(f^{-1}(x)\right) = x$
	$b^{\log_b(x)} = x$	$e^{ln(x)} = x$	Inverse function $f^{-1}\left(f(x)\right) = x$
Exponential Rule	$\log_b(x^r) = r\log_b(x)$ $\log_b\left(\sqrt[y]{x}\right) = \frac{1}{y}\log_b(x)$		$(a^n)^m = a^{nm}$ Convert $\sqrt[y]{x}$ to $x^{\frac{1}{y}}$ first
Product Rule	$\log_b(xy) = \log_b(x) + \log_b(y)$		$a^n a^m = a^{n+m}$ Basis of slide rules
Quotient Rule	$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$		$\frac{a^n}{a^m} = a^{n-m}$
Change of Base	$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$		c is the new base TI-84: [MATH] [MATH] [A:logBASE(]
Base Switch	$\log_b a = \frac{1}{\log_a b}$		Can use instead of TI-84 function
Obscure Rules	$x^{\log_b y} = y^{\log_b x}$ $\log_b(a \pm c) = \log_b(a) + \log_b\left(1 \pm \frac{c}{a}\right)$		Useful in probability theory
Domain	The domain of $\log_b(x)$ is $x > 0$		

Factoring Formulas

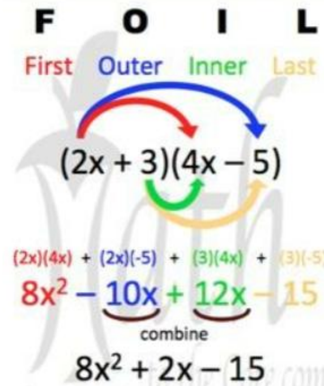
<table>
<tr><th>Description</th><th>Equation</th><th>Notes</th></tr>
<tr><td>Quadratic</td><td>

$x^2 + a^2 = (x + ai)\,(x - ai)$
$x^2 - a^2 = (x + a)\,(x - a)$

$(x + a)^2 = x^2 + 2ax + a^2$
$(x - a)^2 = x^2 - 2ax + a^2$

$(x + a)\,(x + b) = x^2 + (a + b)x + ab$
$(x - a)\,(x - b) = x^2 - (a + b)x + ab$

</td><td>



</td></tr>
<tr><td>Cubic</td><td>

$(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$
$(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$

$x^3 + a^3 = (x + a)\,(x^2 - ax + a^2)$
$x^3 - a^3 = (x - a)\,(x^2 + ax + a^2)$

</td><td>

Binomial expansion (Pascal's Triangle)

Notice one '−' and two '+' for both

</td></tr>
<tr><td>Even Powers</td><td>

$x^{2n} + a^{2n} = (a^n - ix^n)\,(a^n + ix^n)$
$x^{2n} - a^{2n} = (x^n - a^n)\,(x^n + a^n)$

</td><td>The top equation is not often used since it requires imaginary numbers (i)</td></tr>
<tr><td>Odd Powers</td><td colspan="2">

$x^n + a^n = (x + a)\,(x^{n-1} - ax^{n-2} + a^2x^{n-3} - \cdots - a^{n-2}x + a^{n-1})$
$x^n - a^n = (x - a)\,(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \cdots + a^{n-2}x + a^{n-1})$

</td></tr>
<tr><td>End-Terms Method</td><td>

$2x^2 + 13x + 15$

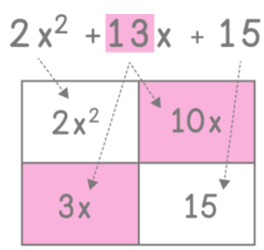
</td><td>

$(\mathbf{2}x \pm 3|5)\,(x \pm \mathbf{5}|3)$

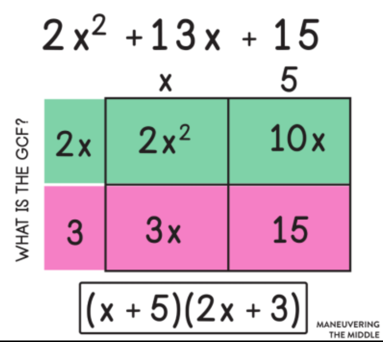
Four possibilities to get middle-term:

1. $+3$ Select one 3
2. -3
3. $+5$ Select one 5
4. -5

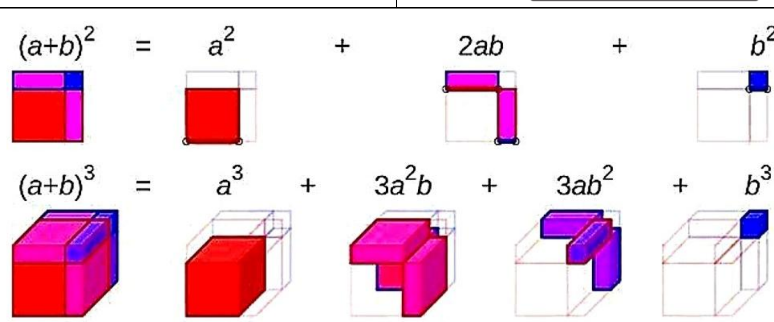
</td></tr>
<tr><td>Box Method</td><td>



</td><td>



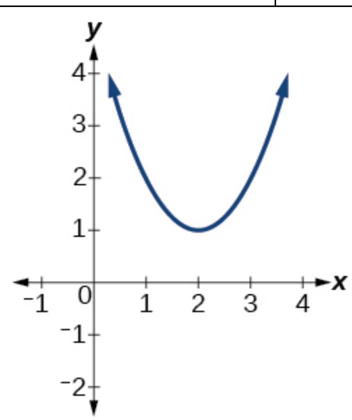
</td></tr>
<tr><td>Visual Method</td><td colspan="2">



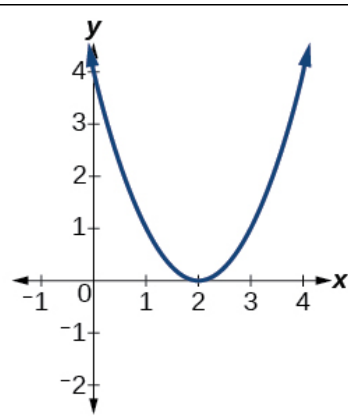
</td></tr>
</table>

Quadratic Formula (x^2)

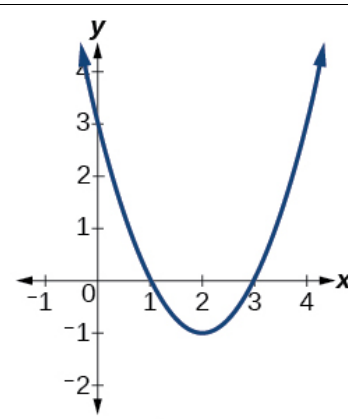
<table>
<tr><th>Description</th><th>Equation</th><th>Notes</th></tr>
<tr><td>Quadratic Formula Form</td><td>$ax^2 + bx + c = 0$</td><td>Remember, b and c can be negative.</td></tr>
<tr><td>Quadratic Formula</td><td>$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$</td><td>• $\sqrt{b^2 - 4ac} > 0$: two real unequal roots
• $\sqrt{b^2 - 4ac} = 0$: repeated real roots (multiplicity of two)
• $\sqrt{b^2 - 4ac} < 0$: two complex roots</td></tr>
<tr><td>Completing the Square</td><td colspan="2">1. Start with the quadratic formula form
$$ax^2 + bx + c = 0$$
2. Divide both sides by a
$$x^2 + \left(\frac{b}{a}\right)x + \left(\frac{c}{a}\right) = 0$$
3. Subtract the constant on the left to move it to the right side
$$x^2 + \left(\frac{b}{a}\right)x = -\left(\frac{c}{a}\right)$$
4. Cut the middle term in half, square it, then add it to both sides
$$x^2 + \left(\frac{b}{a}\right)x + \left(\frac{\mathbf{b}}{\mathbf{2a}}\right)^{\mathbf{2}} = -\left(\frac{c}{a}\right) + \left(\frac{\mathbf{b}}{\mathbf{2a}}\right)^{\mathbf{2}}$$
5. Use an identity to factor the left side
$$a^2 + 2ab + b^2 = (a+b)^2$$
$$\left[x + \left(\frac{b}{2a}\right)\right]^2 = \left(\frac{b}{2a}\right)^2 - \left(\frac{c}{a}\right)\left(\frac{\mathbf{4a}}{\mathbf{4a}}\right)$$
6. Take the square root of both sides
$$x + \left(\frac{b}{2a}\right) = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$
7. Solve for x by subtracting the constant on the left to move it to the right
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$</td></tr>
<tr><td rowspan="2">Vertex</td><td colspan="2">Vertex: $x = -b/2a$
Finds the minimum / maximum of a parabola.</td></tr>
<tr><td colspan="2">$$\frac{d}{dx}[cx^n] = cnx^{n-1}$$
Calculus method: Power Rule</td></tr>
</table>



No x-intercept



One x-intercept

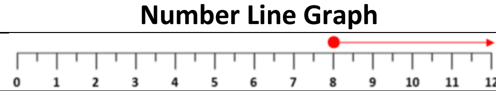
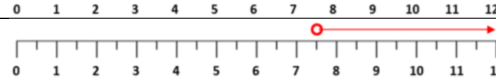
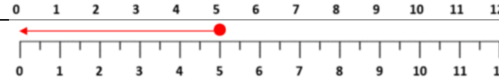
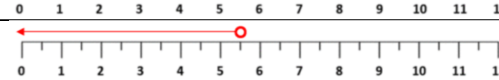
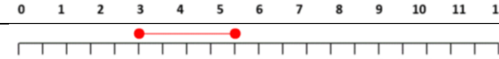
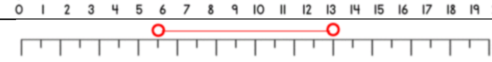
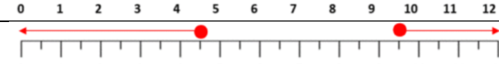
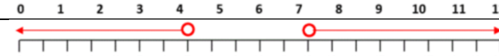


Two x-intercepts

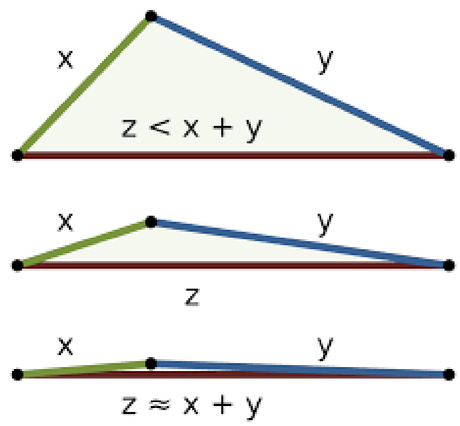
Cubic Formula (x^3)

Description	Equation	Notes
Cubic Formula Form	$ax^3 + bx^2 + cx + d = 0$	Coefficients must be integers
Cubic Formula	$x_1 = \sqrt[3]{e + \sqrt{e^2 + f^3}} + \sqrt[3]{e - \sqrt{e^2 + f^3}} - \frac{b}{3a}$ where $e = \left(\frac{bc}{6a^2} - \frac{d}{2a} - \frac{b^3}{27a^3}\right)$ $f = \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)$ Find x_2 and x_3 using the quadratic formula.	

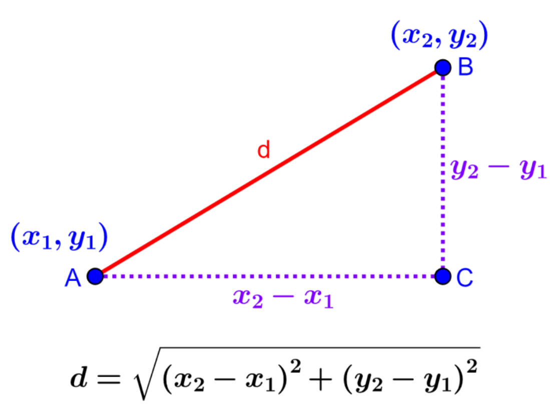
Inequalities ($a > b$)

Property	Equation	Notes
Inequality Symbols	$<, \leq, =, \neq, \geq, >$	$\neq$: <>, ! =
Sum	$a < b$ $\Rightarrow a + c < b + c$	
Difference	$a < b$ $\Rightarrow a - c < b - c$	
Product (Positive)	$a < b \text{ and } c > 0$ $\Rightarrow ac < bc$	Multiplying or dividing by a <u>positive</u> number does not change the inequality direction
Quotient (Positive)	$a < b \text{ and } c > 0$ $\Rightarrow \frac{a}{c} < \frac{b}{c}$	
Product (Negative)	$a < b \text{ and } c < 0$ $\Rightarrow ac > bc$	Multiplying or dividing by a <u>negative</u> number changes the inequality direction (e.g., $c = -1$)
Quotient (Negative)	$a < b \text{ and } c < 0$ $\Rightarrow \frac{a}{c} > \frac{b}{c}$	
Inequality Notation	**Interval Notation**	**Number Line Graph**
$a \leq x$	$[a, \infty)$	
$a < x$	(a, ∞)	
$a \geq x$	$(-\infty, a]$	
$a > x$	$(-\infty, a)$	
$a \leq x \leq b$	$[a, b]$	
$a < x < b$	(a, b)	
$a \geq x \geq b$	$(-\infty, a] \cup [b, \infty)$	
$a > x > b$	$(-\infty, a) \cup (b, \infty)$	

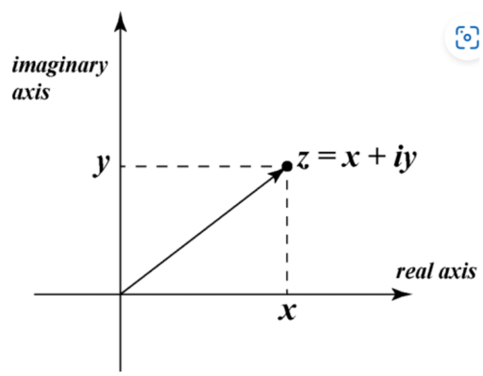
Absolute Values ($|a|$)

Property	Equation	Notes
Definition	$\|a\| = \begin{cases} a & if\ a \geq 0 \\ -a & if\ a < 0 \end{cases}$	Simply strip away the minus sign
Observations	$\|a\| \geq 0$	Are never negative
	$\|-a\| = \|a\|$	We care about the magnitude only
Product Rule	$\|ab\| = \|a\|\,\|b\|$	
Quotient Rule	$\left\|\frac{a}{b}\right\| = \frac{\|a\|}{\|b\|}$	
Solving	$\|ax + b\| = c$	Positive Case: $(ax + b) = c$ Negative Case: $-(ax + b) = c$
Triangle Inequality	$\|a + b\| \leq \|a\| + \|b\|$ $\|a - b\| \geq \|\|a\| - \|b\|\|$	

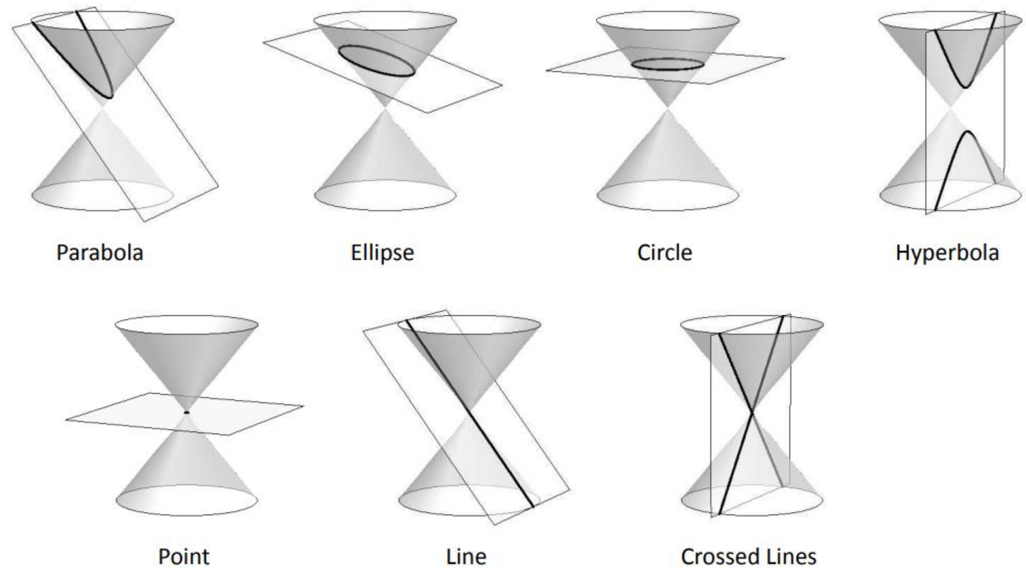
Distance Formula

Property	Equation
Distance Formula	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Same as Pythagorean Theorem $c^2 = a^2 + b^2$
Graph	

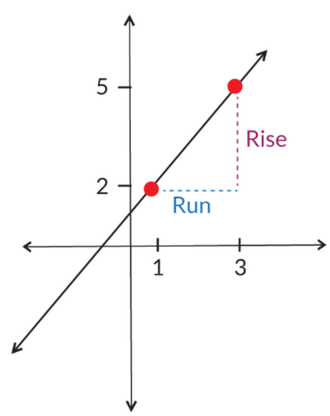
Complex Numbers ($a + bi$)

Property	Equation	Notes
Imaginary Number (Adjacent Number)	$i = \sqrt{-1}$ $i^2 = -1$ $i^3 = -i = -\sqrt{-1}$ $i^4 = 1$	$i^{4n} = 1$ $i^{4n+1} = i$ $i^{4n+2} = -1$ $i^{4n+3} = -i$
Usage	$\sqrt{-a} = i\sqrt{a}$	Assuming $a \geq 0$
Complex Number	$a + bi$	Real + Imaginary
Sum	$(a + bi) + (c + di) = a + c + (b + d)i$	
Difference	$(a + bi) - (c + di) = a - c + (b - d)i$	
Product	$(a + bi)\,(c + di) = ac - bd + (ad + bc)i$	
Quotient	$\frac{c}{(a + bi)} = \frac{c}{(a + bi)} \frac{(\boldsymbol{a} - \boldsymbol{bi})}{(\boldsymbol{a} - \boldsymbol{bi})} = \frac{c(a - bi)}{a^2 + b^2}$	
Conjugate	$\overline{(a + bi)} = a - bi$	$\overline{(a - bi)} = a + bi$
Modulus	$\lvert a + bi \rvert = \sqrt{a^2 + b^2}$	Distance from origin $(0, 0)$ to (a, b)
Squares	$a^2 + b^2 = (a + bi)\,(a - bi)$ $\lvert a + bi \rvert^2 = (a + bi)\,(a - bi)$	
Euler's Identity	$e^{i\pi} + 1 = 0$	
Graph		

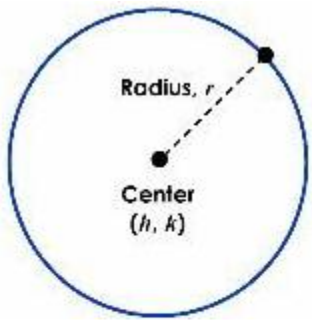
Conic Sections



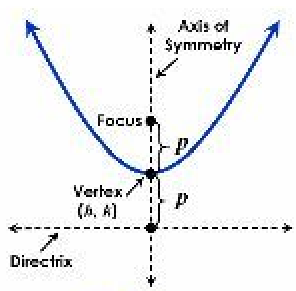
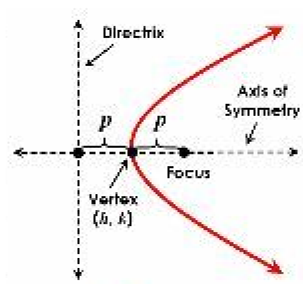
Line

Description	Equation	Notes
Slope	$m = \frac{rise}{run}$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$	$P_1 = (x_1, y_1)$ $P_2 = (x_2, y_2)$ Passes through points P_1 and P_2
Standard Form	$ax + by + c = 0$	Where a is positive
Slope-Intercept Form	$y = mx + b$ $f(x) = mx + b$	b is the y-axis intercept Passes through point $(0, b)$
Point-Slope Form	$y - y_0 = m(x - x_0)$	Manipulation of the slope formula
Intercept Form	$\frac{x}{a} + \frac{y}{b} = 1$	a is the x-intercept b is the y-intercept
Vertical Line	$x = a$	Passes through point $(a, 0)$
Horizontal Line	$y = b$ $f(x) = b$	Passes through point $(0, b)$
Domain	The domain of $y = mx + b$ is $(-\infty, \infty)$	

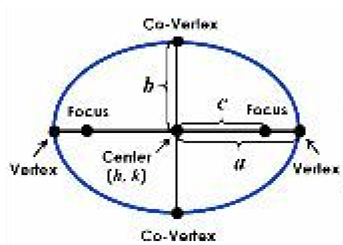
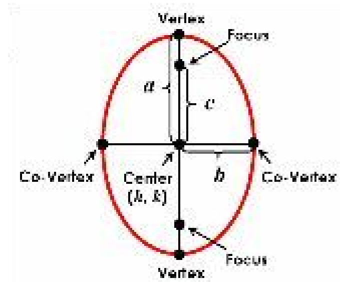
Circle

Description	y-Axis Symmetry	x-Axis Symmetry
Standard Form	$x^2 + y^2 = r^2$ $(x-h)^2 + (y-k)^2 = r^2$	Same
Radius	r $r = \sqrt{(x_1-h)^2 + (y_1-k)^2}$	Same
Center	(h, k)	Same
Vertices	None	Same
Focus	(h, k)	Same
Graph		Same

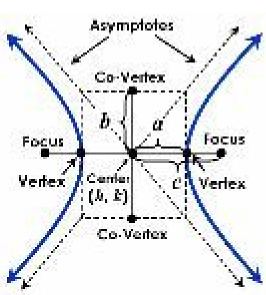
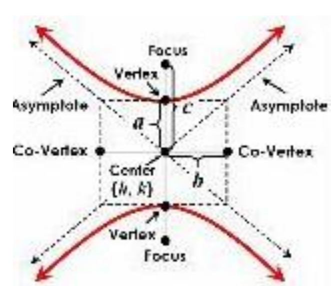
Parabola

Description	y-Axis Symmetry	x-Axis Symmetry
Standard Form	$y = ax^2 + bx + c$ $f(x) = ax^2 + bx + c$	$x = ay^2 + by + c$ $g(y) = ay^2 + by + c$
Vertex Form	$y = a(x-h)^2 + k$	$x = a(y-k)^2 + h$
Factored Form	$y = a(x-r_1)(x-r_2)$	Roots r_1, r_2
Graphing Form	$x^2 = 4py$ $(x-h)^2 = 4p(y-k)$	$y^2 = 4px$ $(y-k)^2 = 4p(x-h)$
Vertex	(h, k) $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$	(h, k) $\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right)$
Focus	$(h, k+p)$	$(h+p, k)$
Directrix	$y = k - p$	$x = h - p$
Direction	Opens up if $a > 0$ Opens down if $a < 0$	Opens right if $a > 0$ Opens left if $a < 0$
Graph		

Ellipse

Description	y-Axis Symmetry	x-Axis Symmetry
Standard Form	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$ $\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$
Center	(h,k)	(h,k)
Vertices	$(h\pm a,k)$	$(h,k\pm a)$
Co-Verticies	$(h,k\pm b)$	$(h\pm b,k)$
Foci	$(h\pm c,k)$	$(h,k\pm c)$
Focus Length	$c^2 = a^2-b^2$	$c^2 = a^2-b^2$
Graph		
The <u>sum</u> of the distances from each focus to a point on the curve is constant. $\lvert d_1+d_2\rvert = k$		

Hyperbola

Description	y-Axis Symmetry	x-Axis Symmetry
Standard Form	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Center	(h,k)	(h,k)
Vertices	$(h\pm a,k)$	$(h,k\pm a)$
Foci	$(h\pm c,k)$	$(h,k\pm c)$
Focus Length	$c^2 = a^2+b^2$	$c^2 = a^2+b^2$
Asymptotes	$y=k\pm\left(\frac{b}{a}\right)(x-h)$	$y=k\pm\left(\frac{a}{b}\right)(x-h)$
Direction	Opens left/right	Opens up/down
Graph		
The <u>difference</u> between the distances from each focus to a point on the curve is constant. $\lvert d_1-d_2\rvert = k$		

Sources:

- Brack, Tyne (2023, 24 January). Maneuvering the Middle, Factoring Polynomials with Special Cases. https://www.maneuveringthemiddle.com/factoring-polynomials-with-special-cases/

- Dawkins, Paul (2024). Paul's Online Notes, Algebra Cheat Sheet. https://tutorial.math.lamar.edu

- Larson, Ron & Edwards, Bruce (2009). Algebra 1-Pager Reference, Calculus 9th Edition. https://www.amazon.com/Bruce-Edwards-Larson-Calculus-Hardcover/dp/B004QICACS/ref=sr_1_1

- Math Salamanders (2024). Inequalities on a Number Line. https://www.math-salamanders.com/inequalities-on-a-number-line.html

- TeachersPayTeachers.com (2019). Conic Section Graphs, All Things Algebra. https://www.pinterest.com/pin/612911830514800210/

ALGEBRA

Factors and Zeros of Polynomials

Let $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial. If $p(a) = 0$, then a is a *zero* of the polynomial and a solution of the equation $p(x) = 0$. Furthermore, $(x - a)$ is a *factor* of the polynomial.

Fundamental Theorem of Algebra

An nth degree polynomial has n (not necessarily distinct) zeros. Although all of these zeros may be imaginary, a real polynomial of odd degree must have at least one real zero.

Quadratic Formula

If $p(x) = ax^2 + bx + c$, and $0 \le b^2 - 4ac$, then the real zeros of p are $x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/2a$.

Special Factors

$x^2 - a^2 = (x - a)(x + a)$

$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

$x^4 - a^4 = (x^2 - a^2)(x^2 + a^2)$

Binomial Theorem

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \cdots + nxy^{n-1} + y^n$$

$$(x - y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 - \cdots \pm nxy^{n-1} \mp y^n$$

Rational Zero Theorem

If $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients, then every *rational zero* of p is of the form $x = r/s$, where r is a factor of a_0 and s is a factor of a_n.

Factoring by Grouping

$$acx^3 + adx^2 + bcx + bd = ax^2(cx + d) + b(cx + d) = (ax^2 + b)(cx + d)$$

Arithmetic Operations

$ab + ac = a(b + c)$

$\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + bc}{bd}$

$\dfrac{a + b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$

$\dfrac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \left(\dfrac{a}{b}\right)\left(\dfrac{d}{c}\right) = \dfrac{ad}{bc}$

$\dfrac{\left(\frac{a}{b}\right)}{c} = \dfrac{a}{bc}$

$\dfrac{a}{\left(\frac{b}{c}\right)} = \dfrac{ac}{b}$

$a\left(\dfrac{b}{c}\right) = \dfrac{ab}{c}$

$\dfrac{a - b}{c - d} = \dfrac{b - a}{d - c}$

$\dfrac{ab + ac}{a} = b + c$

Exponents and Radicals

$a^0 = 1, \quad a \ne 0$

$(ab)^x = a^x b^x$

$a^x a^y = a^{x+y}$

$\sqrt{a} = a^{1/2}$

$\dfrac{a^x}{a^y} = a^{x-y}$

$\sqrt[n]{a} = a^{1/n}$

$\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

$\sqrt[n]{a^m} = a^{m/n}$

$a^{-x} = \dfrac{1}{a^x}$

$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

$(a^x)^y = a^{xy}$

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$