Harold's Ordinary Differential Equations (ODE) Cheat Sheet

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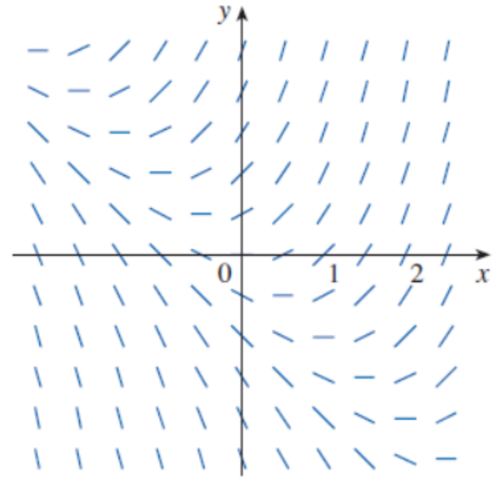
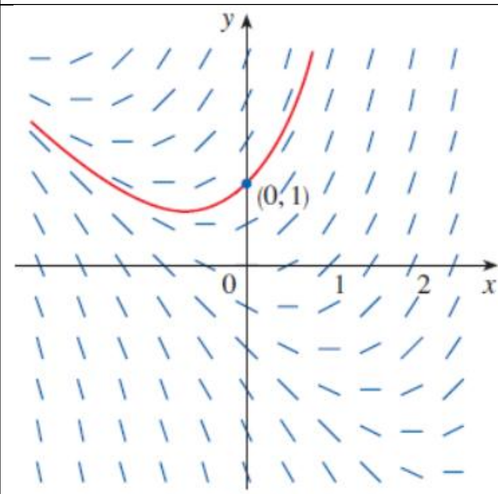
Classification

Term	Definition
By Type	
Differential Equation (DiffEq)	A mathematical equation that relates a function to its derivatives.
Ordinary Differential Equation (ODE)	A differential equation that involves an unknown function and its derivatives with respect to a single independent variable. For example: $$\frac{dy}{dx} = x^2 + 3x$$
Partial Differential Equation (PDE)	A type of equation that involves an unknown function and its partial derivatives with respect to multiple independent variables. For example: $$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$
Systems of Differential Equations	Uses vector notation. For example: $$x' = Ax$$
By Order	
Order	The order of the highest derivative in the equation. y' = 1st order y'' = 2nd order y''' = 3rd order $y^{(4)}$ = 4th order
By Linearity	
General Solution	A family of functions that has parameters and does not take any initial conditions into account.
Particular Solution (Actual Solution)	Has no arbitrary parameters and satisfies the initial conditions.
Singular Solution	Cannot be obtained from the general solution.
Linear Equation	$$\boldsymbol{y^{(n)}(x) + a_{n-1}(x)y^{(n-1)}(x) + \cdots + a_1(x)y'(x) + a_0(x)y(x) = f(x)}$$ $$where\ each\ a_i(x)\ is\ a\ function\ of\ x.$$
Nonlinear Equation	An equation that contains functions of y, such as e^y, or functions of the derivatives of y, such as $(5y)y'', sin(y'), or\ \left(\frac{dy}{dx}\right)^2$.

Terms and Notation

Notation	Expanded Form	Description
$\boldsymbol{x, y, t}$	x, y, t	Variables for space and time
$\boldsymbol{f}$	$f = f(x)$ or y	The function of a single independent variable.
$\boldsymbol{f'}$	$\frac{df}{dx}, \frac{dy}{dx}, or\ y'$	The derivative of f with respect to x.
$\boldsymbol{f_x}$	$\frac{\partial f}{\partial x}$	The partial derivative of f with respect to x.
$\boldsymbol{y_p}$	Particular solution to the equation.	
IVP	Initial value problem. An initial condition that specifies the value of the unknown function at a given point in the domain so a particular solution can be found. e.g., $y(0) = 1, y'(0) = -1$	
$\boldsymbol{\Gamma(t)}$	Gamma Function	The Gamma function is the continuous version of the discrete factorial function, $n!$. $\Gamma(t) = \int_0^\infty e^{-x} x^{t-1}\ dx$ $\Gamma(n+1) = n!$

Direction Fields

Name	Slope Equation	Graph
Direction Field (Slope Field)	$y' = x + y$ **General** Solution: $y(x) = c_1 e^x - x - 1$	
Solution Curve	$\frac{dy}{dx} = x + y$ Initial Condition: $y(0) = 1$ **Particular** Solution: $y(x) = 2e^x - x - 1$	

1st-Order Linear

Technique	Form of Equation	Solution Method
1st-Order	$F(x,y,y') = 0$ $y' + a(x)y = f(x)$	$I.F. = e^{\int a(x)\,dx}$ $y = Ce^{-\int a(x)\,dx}$
Single Variable	$\frac{dy}{dx} = f(x)$	$y(x) = \int f(x)\ dx + C$
Separable Equations (Separation of Variables)	$\frac{dy}{dx} = f(x,y) = \frac{g(x)}{f(y)}$	$\int f(y)\,dy = \int g(x)\,dx + C$
Integration Factor ($\boldsymbol{I(x,y) = \mu(x)}$)	$\frac{dy}{dx} + P(x)y = Q(x)$ $\mu(x) = e^{\int P(x)\,dx}$	1. $\mu(x) = e^{\int P(x)\,dx}$ 2. Multiplying the entire equation by $\mu(x)$. $\mu(x)\frac{dy}{dx} + \mu(x)P(x)y = \mu(x)Q(x)$ 3. But the derivative of the product is: $\frac{d(\mu(x)y(x))}{dx} = \mu(x)\frac{dy}{dx} + \mu(x)P(x)y$ 4. $\mu(x)y(x) = \int \mu(x)\,Q(x)\,dx + C$ 5. $\boldsymbol{y(x) = \frac{\int \mu(x)\,Q(x)\,dx + C}{\mu(x)}}$
Exact Equations	$M(x,y)\,dx + N(x,y)\,dy = 0$ $= dg(x.y)$ $Iff\ \ \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$	Find $g(x,y)$ by integrating and comparing: $\int M(x,y)\,dx\ \ and\ \ \int N(x,y)\,dy$
Non-Exact Equations	Reduction to Exact via Integrating Factor. $\boldsymbol{I(x,y)\left[M(x,y) + N(x.y)\frac{dy}{dx}\right] = 0}$	
	If	**Then**
Case 1: $\boldsymbol{\mu(x)}$	$\mu_1(x) \equiv \frac{M_y - N_x}{N}$	$I(x,y) = e^{\int \mu_1(x)\,dx}$
Case 2: $\boldsymbol{\mu(y)}$	$\mu_2(y) \equiv \frac{N_x - M_y}{M}$	$I(x,y) = e^{\int \mu_2(y)\,dy}$
Case 3: $\boldsymbol{\mu(xy)}$	$\mu_3(xy) \equiv \frac{x(N_x - M_y)}{yM - xN}$	$I(x,y) = e^{\int \mu_3(xy)\,d(xy)}$
Bernoulli	$\frac{dy}{dx} + P(x)y = Q(x)y^n$	If $n = 0$ or $n = 1$, this is linear, else 1. Divide by $\frac{1}{y^n}$ 2. Change variable $z = \frac{1}{y^{1-n}}$ 3. Apply substitution to get a linear form 4. Use an integrating factor to solve

Homogeneous	$M(x,y)\,dx + N(x,y)\,dy = 0$ where $M(x,y)$ and $N(x,y)$ are homogeneous functions of the same degree n, meaning: $M(\lambda x, \lambda y) = \lambda^n M(x,y)$ $N(\lambda x, \lambda y) = \lambda^n N(x,y)$	1. Start with: $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ 2. Reduce to separation of variables using: $v = \frac{y}{x}$ $y = vx$ 3. Product rule: $\frac{dy}{dx} = v + x\frac{dv}{dx}$ 5. Rewrite in terms of $\frac{dv}{dx}$.
Substitution	$\frac{dy}{dx} = F(ax + by + c)$	1. Set $v = ax + by + c$ 2. Solve for y 3. Find $\frac{dy}{dx}$ in terms of $\frac{dv}{dx}$ 4. Rewrite the equation in terms of $\frac{dv}{dx}$
Reduction by Translation	$\boldsymbol{y' = \frac{Ax + By + C}{Dx + Ey + F}}$	
	If	**Then**
Case 1: Intersecting Lines	$A \neq B$ $D \neq E$	1. Put $x = X + h \text{ and } y = Y + k$ 2. Find h and k 3. Solve using separation of variables 4. Translate back
Case 2: Parallel Lines	$A = B$ $D = E$	1. Put $u = Ax + By$ $y' = \frac{u' - A}{B}$ 2. Solve

2nd-Order Linear Homogeneous ($=0$)

Technique	Form of Equation	Solution Method
2nd-Order	$F(x, y, y', y'') = \mathbf{0}$	Typical case has all constant (a, b, c) coefficients instead of functions of x.
Homogeneous	$P(x)y'' + Q(x)y' + R(x)y = \mathbf{0}$ $ay'' + by' + cy = \mathbf{0}$	$y(x) = c_1 y_1(x) + c_2 y_2(x)$
Principle of Superposition	If $y'' + ay' + by = f_1(x)$ has solution $y_1(x)$ and $y'' + ay' + by = f_2(x)$ has solution $y_2(x)$	then $y'' + ay' + by = f(x) = f_1(x) + f_2(x)$ has solution $\boldsymbol{y(x) = c_1 y_1(x) + c_2 y_2(x)}$
Characteristic Equation (Constant Coefficients)	$\boldsymbol{ay'' + by' + cy = 0}$ $\Downarrow$ $\boldsymbol{ar^2 + br + c = 0}$	All solutions are in the form: $y(x) = e^{rx}$ Roots: r_1, r_2
	If	**Then**
Case 1: Distinct Roots	$r_1, r_2 \in \mathbb{R}$ $r_1 \neq r_2$	1. Set $y_1(x) = e^{r_1 x}$ $y_2(x) = e^{r_2 x}$ 2. Superposition gives $\boldsymbol{y(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x}}$
Case 2: Repeated Root	$r_1, r_2 \in \mathbb{R}$ $r_1 = r_2 = r$	$\boldsymbol{y(x) = c_1 e^{rx} + c_2 x e^{rx}}$
Case 3: Complex Roots	$r_{1,2} = \lambda \pm \mu\, i$ with $\lambda = -\dfrac{b}{2a}, \qquad \mu = \dfrac{\sqrt{4ac - b^2}}{2a}$	1. Set $y_1(x) = e^{(\lambda + \mu\, i)x}$ $y_2(x) = e^{(\lambda - \mu\, i)x}$ 2. Superposition gives $y(x) = c_1 e^{(\lambda + \mu\, i)x} + c_2 e^{(\lambda - \mu\, i)x}$ 3. Apply $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ 4. $\boldsymbol{y(x) = c_1 e^{\lambda x}\cos(\mu x) + c_2 e^{\lambda x}\sin(\mu x)}$
Reduction of Order	$P(x)y'' + Q(x)y' + R(x)y = 0$	If we already know y_1, 1. Put $y_2 = v y_1$ 2. Expand in terms of v'', v', v 3. Put $z = v'$ 4. Solve the reduced equation
Wronskian (Linear Independence)	$y_1(x)$ and $y_2(x)$ are linear independent iff	$W(y_1, y_2)(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$
Euler-Cauchy Equation	$x^2 y'' + axy' + by = 0$ where $x \neq 0$ $A.E.: \lambda(\lambda - 1) + a\lambda + b = 0$	1. $y(x)$ of the form x^λ 2. Reduction to Constant Coefficients: Use $x = e^t, t = \ln x$ 3. Rewrite in terms of t using chain rule.
Case 1: Distinct Roots	$\lambda_1, \lambda_2 \in \mathbb{R}$ $\lambda_1 \neq \lambda_2$ $x \neq 0$	$y(x) = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

Case 2: Repeated Root	$\lambda_1, \lambda_2 \in \mathbb{R}$ $\lambda_1 = \lambda_2 = \lambda$	$y(x) = x^{\lambda}(c_1 + c_2 \ln\lvert x\rvert)$
Case 3: Complex Roots	$\lambda_{1,2} = \alpha \pm i\beta$	$y(x) = x^{\alpha}(c_1 \cos(\beta \ln\lvert x\rvert)$ $+ c_2 \sin(\beta \ln\lvert x\rvert))$

2nd-Order Linear Non-Homogeneous ($= \boldsymbol{f(x)}$)

Technique	Form of Equation	Solution Method
2nd-Order	$F(x, y, y', y'') = \boldsymbol{f(x)}$ $y'' + a(x)y' + b(x)y = \boldsymbol{f(x)}$	$y = y_h + y_p$ $= \boldsymbol{c_1 y_1(x) + c_2 y_2(x) + y_p(x)}$ where the general solution includes the particular solution, y_p.
Non-Homogeneous	$P(x)y'' + Q(x)y' + R(x)y$ $= \boldsymbol{f(x)}$ $ay'' + by' + cy = \boldsymbol{f(x)}$ $f(x) \neq 0$	If homogeneous, then $f(x) = 0$.
Case 1: $\boldsymbol{y', y}$ Missing	$y'' = f(x)$	$y(x) = \iint f(x)\ dx$
Case 2: $\boldsymbol{y}$ Missing	$y'' = f(y', x)$	1. Change of variable: $p = y'$ 2. Solve twice
Case 3: $\boldsymbol{x}$ Missing	$y'' = f(y', y)$	1. Change of variable: $p = y'$ + chain rule 2. $p\frac{dp}{dy} = f(p, y)$ is a 1st-order ODE 3. Solve it 4. Back-replace p 5. Solve again
Case 4: $\boldsymbol{y', x}$ Missing	$y'' = f(y)$	1. Change of variable: $p = y'$ + chain rule 2. $p\frac{dp}{dy} = f(y)$ is a separation of variables 3. Solve it 4. Back-replace p 5. Solve again
Method of Undetermined Coefficients (Guesswork)	Assume $y(x)$ has the same form as $f(x)$ with undetermined constant coefficients. Valid forms: 1. $P_n(x)$ 2. $P_n(x)\, e^{ax}$ 3. $e^{ax}\, (P_n(x) \cos bx + Q_n(x) \sin bx)$ Failure case: If any term of $f(x)$ is a solution of y_b, multiply y_p by x and repeat until it works.	

Laplace Transforms

Technique	Equation	Solution
Laplace Transform	$\boldsymbol{\mathcal{L}\{f(t)\} = F(s)}$ • An integral transform that converts a function of a real variable (usually t, in the time domain) to a function of a complex variable s in the complex-valued frequency domain (s domain). ○ NOTE: Frequency $s = 1/t$ • Transforms ODEs into algebraic equations. • The substitution $x = e^{-s}$ makes the integral look like a formal power series, with $f(t)$ being the coefficient of x^t. • The z-transform is the discrete analog of the Laplace transform in signal processing.	
Inverse Laplace Transform	$\boldsymbol{\mathcal{L}^{-1}\{F(s)\} = f(t)}$	
Laplace Transform Equation	$$\boldsymbol{\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-sx} f(t)\ dt}$$ where • $f(t)$ must be defined on $0 \leq t \leq \infty$ • s is an arbitrary real variable • The improper integral must converge	
Transform of a Derivative	**Theorem**: If $f(t), f'(t), \dots, f^{(n-1)}(t)$ are continuous on [0,∞) and are of exponential order, and if $f^{(n)}(t)$ is piecewise continuous on [0,∞), then the Laplace transform of the nth derivative of $f(t)$ is given by the general nth order derivative equation below.	
Laplace Transform of Derivatives	First-order derivative	$\mathcal{L}\{f'(t)\} = s\,\mathcal{L}\{f(t)\} - f(0)$
	Second-order derivative	$\mathcal{L}\{f''(t)\} = s^2\mathcal{L}\{f(t)\} - s\,f(0) - f'(0)$
	Third-order derivative	$\mathcal{L}\{f'''(t)\} = s^3\mathcal{L}\{f(t)\} - s^2 f(0) - s\,f'(0) - f''(0)$
	n^{th} order derivative	$\mathcal{L}\{f^{(n)}(t)\} = s^n\mathcal{L}\{f(t)\} - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$
Using Laplace to Solve a DiffEq	Solves IVPs for linear differential equations with constant coefficients. 1. Apply the Laplace transform to each term in the IVP. 2. Solve the resulting algebraic equation for the transformed function $F(s)$. 3. Apply the inverse Laplace transform to find the solution.	

Table of Laplace Transforms ($s > 0$)

#	$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1	$f'(t)$	$sF(s) - f(0)$
2	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
3	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$
4	$f^{(4)}(t)$	$s^4F(s) - s^3f(0) - s^2f'(0) - sf''(0) - f'''(0)$
5	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
6	1	$\frac{1}{s}$
7	e^{at}	$\frac{1}{s-a}$
8	te^{at}	$\frac{1}{(s-a)^2}$
9	$t^n e^{at}$ $(n = 1, 2, 3, \ldots)$	$\frac{n!}{(s-a)^{n+1}}$
10	$\frac{e^{at} - e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}$
11	$\frac{ae^{at} - be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$
12	t	$\frac{1}{s^2}$
13	t^n $(n = 1, 2, 3, \ldots)$	$\frac{n!}{s^{n+1}}$
14	t^p $(p \geq -1 \in \mathbb{R})$	$\frac{\Gamma(p+1)}{s^{p+1}}$
15	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$
16	$t^{n-\frac{1}{2}}$ $(n = 1, 2, 3, \ldots)$	$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n\, s^{n+\frac{1}{2}}}$
17	$sin(at)$	$\frac{a}{s^2 + a^2}$
18	$cos(at)$	$\frac{s}{s^2 + a^2}$
19	$sin\,(at + b)$	$\frac{s\,sin(b) + a\,cos\,(b)}{s^2 + a^2}$
20	$cos\,(at + b)$	$\frac{s\,cos(b) - a\,sin\,(b)}{s^2 + a^2}$
21	$t\,sin(at)$	$\frac{2as}{(s^2 + a^2)^2}$
22	$t\,cos(at)$	$\frac{s^2 - a^2}{(s^2 + a^2)^2}$
23	$\frac{1}{t}\sin(at)$	$\arctan\left(\frac{a}{s}\right)$
24	$e^{at}\,sin\,(bt)$	$\frac{b}{(s-a)^2 + b^2}$

25	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
26	$\sin(at) - at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
27	$\sin(at) + at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
28	$\cos(at) - at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$
29	$\cos(at) + at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
30	$\sinh(at)$	$\frac{a}{s^2-a^2}$
31	$\cosh(at)$	$\frac{s}{s^2-a^2}$
32	$t\sinh(at)$	$\frac{2as}{(s^2-a^2)^2}$
33	$t\cosh(at)$	$\frac{s^2-a^2}{(s^2-a^2)^2}$
34	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$
35	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
36	$e^{at}f(t)$	$F(s-a)$
37	$tf(t)$	$-F'(s)$
38	$t^n f(t)$ $(n = 1, 2, 3, \dots)$	$(-1)^n F^{(n)}(s)$
39	$\frac{1}{t} f(t)$	$\int_s^{\infty} F(u)\,du$
40	$f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right)$
41	$u_a(t) = u(t-a)$ Heaviside Function	$\frac{1}{s} e^{-as}$
42	$\delta(t)$	1
43	$\delta(t-a)$ Dirac Delta Function	e^{-as}
44	$u_a(t)\,f(t-a)$	$e^{-as}\,F(s)$
45	$u_a(t)\,g(t)$	$e^{-as}\,\mathcal{L}\{g(t+a)\}$
46	$\int_0^t f(v)\,dv$	$\frac{1}{s} F(s)$
47	$\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
48	$f(t+T) = f(t)$	$\frac{\int_0^T e^{-st} f(t)\,dt}{1-e^{-sT}}$
49	$af(t) + bg(t)$	$aF(s) + bG(s)$
50	$\frac{1}{\sqrt{\pi t}} e^{\frac{-a^2}{4t}}$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$

51	$\frac{a}{2\sqrt{\pi t^3}} e^{\frac{-a^2}{4t}}$	$e^{-a\sqrt{s}}$
52	$erfc\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$

Fourier Transforms

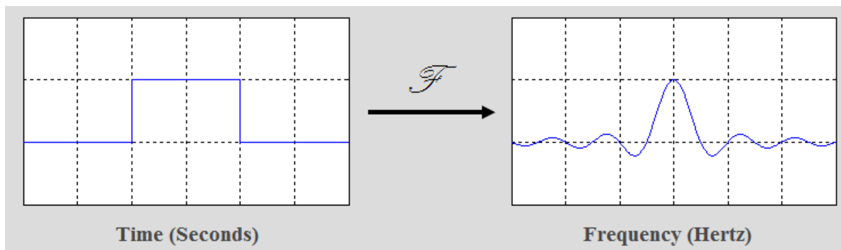
Technique	Equation	Solution
Fourier Series	• A method to **decompose** a periodic signal into a sum of sine and cosine functions with different frequencies and amplitudes. • Is particularly useful for analyzing arbitrary **periodic** signals. • Was introduced by Joseph Fourier. • Is widely used in fields such as electronics, signal processing, and quantum mechanics.	
Fourier Transform	• A mathematical operation that **transforms** a signal from the time domain (t) to the frequency domain (f). • Unlike the Fourier Series, the Fourier Transform can be applied to both **periodic** and **non-periodic** signals. • Are widely used to solve differential equations. 	
Fourier Transform Equation	$$\boldsymbol{\mathcal{F}\{f(t)\} = \hat{f}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt}$$ where • $f(t)$ must be defined on $-\infty \le t \le \infty$ • ω is an arbitrary real variable where $\omega = 2\pi f$ • The improper integral must converge	
Inverse Fourier Transform Equation	$$\mathcal{F}^{-1}\{\hat{f}(\omega)\} = f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega)\, e^{i\omega t}\, d\omega$$ where • $\hat{f}(\omega)$ must be defined on $-\infty \le \omega \le \infty$ • ω is an arbitrary real variable where $\omega = 2\pi f$ • The improper integral must converge	
Example	$$f(t) = \begin{cases} e^{-at} & for\ t > 0 \\ 0 & for\ t < 0 \end{cases}$$ $$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt = \int_{0}^{\infty} e^{-at}\, e^{-it\omega}\, dt = \frac{1}{a + i\omega}$$	
Using Fourier Transforms to Solve a DiffEq	Solves ODEs for linear differential equations with constant coefficients. 1. Apply the Fourier transform to each term in the ODE. 2. Solve the resulting algebraic equation for the transformed function $\hat{f}(\omega)$. 3. Apply the inverse Fourier transform to find the solution.	

Table of Fourier Transforms ($a, b > 0$)

#	$f(x)$	$\mathcal{F}\{f(t)\} = \hat{f}(\omega)$
1	$a\,f(x) + b\,g(x)$	$a\,\hat{f}(\omega) + b\,\hat{g}(\omega)$
2	$f(x-a)$	$e^{-ia\omega}\,\hat{f}(\omega)$
3	$f(x)\,e^{iax}$	$\hat{f}(\omega - a)$
4	$f(ax)$	$\frac{1}{\lvert a\rvert}\hat{f}\left(\frac{\omega}{a}\right)$
5	$\frac{d^n f(x)}{dx^n}$	$(i\omega)^n \hat{f}(\omega)$
6	$x^n f(x)$	$i^n\,\frac{d^n \hat{f}(\omega)}{d\omega^n}$
7	$(f \circ g)(x)$	$\hat{f}(\omega)\,\hat{g}(\omega)$
8	$f(x)\,g(x)$	$\frac{1}{2\pi}\,(f \circ g)(\omega)$
9	$Real\ f(x)$	$\hat{f}(-\omega) = \overline{\hat{f}(\omega)}$
10	$Imaginary\ f(x)$	$\hat{f}(-\omega) = -\overline{\hat{f}(\omega)}$
11	$\overline{f(x)}$	$\overline{\hat{f}(-\omega)}$
12	$f(x)\cos(ax)$	$\frac{\hat{f}(\omega-a) + \hat{f}(\omega+a)}{2}$
13	$f(x)\sin(ax)$	$\frac{\hat{f}(\omega-a) - \hat{f}(\omega+a)}{2i}$
14	$e^{-ax}\,u(x)$	$\frac{1}{a+i\omega}$
15	$e^{-a\lvert x\rvert}$	$\frac{2a}{a^2+\omega^2}$
16	1	$2\pi\delta(\omega)$
17	$\delta(x)$	1
18	$\sin(ax)$	$-i\pi(\delta(\omega-a) + \delta(\omega+a))$
19	$\cos(ax)$	$\pi(\delta(\omega-a) + \delta(\omega+a))$
20	$\sin(ax^2)$	$-\sqrt{\frac{\pi}{a}}\cos\left(\frac{\omega^2}{4a} - \frac{\pi}{4}\right)$
21	$\cos(ax^2)$	$\sqrt{\frac{\pi}{a}}\cos\left(\frac{\omega^2}{4a} - \frac{\pi}{4}\right)$
22	$sech(ax)$	$\frac{\pi}{a}sech\left(\frac{\pi}{2a}\omega\right)$
23	$e^{-\pi i\alpha x^2}$	$\frac{1}{\sqrt{\alpha}}\,e^{-i\frac{\pi}{4}}\,e^{i\frac{\omega^2}{2\pi\alpha}}$
24	x^n	$2\pi i^n\,\delta^{(n)}(\omega)$
25	$\delta^{(n)}(x)$	$(i\omega)^n$
26	$\frac{1}{x}$	$-i\pi\,sgn(\omega)$
27	$\frac{1}{x^n}$	$-i\pi\frac{(-i\omega)^{n-1}}{(n-1)!}sgn(\omega)$
28	$\frac{1}{\sqrt{\lvert x\rvert}}$	$\frac{\sqrt{2\pi}}{\sqrt{\lvert\omega\rvert}}$

Sources

- Dawkins, Paul (2004). Table of Laplace Transforms. https://tutorial.math.lamar.edu/classes/de/Laplace_Table.aspx
- Furius Enterprises (2010). Differential Equations Cheat sheet 2nd-order Homogeneous. https://furius.ca/cqfpub/doc/diffequs/diffequs.pdf
- Shapiro, B.E. (2014). Table of Laplace Transforms. https://www.integral-table.com/downloads/LaplaceTable.pdf
- Wikipedia (2025). Tables of important Fourier transforms. https://en.wikipedia.org/wiki/Fourier_transform#Tables_of_important_Fourier_transforms

See Also

- Harold's Partial Differential Equations (PDE) Cheat Sheet
- Harold's Differential Equation Models Cheat Sheet
- Harold's Euler's Method Example