**Harold’s DiffEq Euler’s Method Example**

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**Problem:**

Use Euler’s Method with h = 0.1 to approximate the solution to the following initial value problem on the interval 1 ≤ x ≤ 2.

Compare these approximations with the actual solution $y=-\frac{1}{x}$ by graphing the polygonal-line approximation and the actual solution on the same coordinate system.

$$y^{'}=\frac{1}{x^{2}}-\frac{y}{x}-y^{2}, y\left(1\right)=-1$$

Graph the polygonal-line approximation and the actual solution on the same coordinate system.

Choose the correct graph below.

   

**Solution:**

Euler’s Method:

 $\frac{dy}{dx}=f\left(x,y\right)$

 $y\left(x\_{0}\right)=y\_{0}$

 $y\_{n+1}=y\_{n}+hf\left(x\_{n}, y\_{n}\right)$

Notice that the 3rd equation above is simply the slope equation

$$m=\frac{∆y}{∆x}=\frac{y\_{n+1}- y\_{n}}{h}≈f\left(x\_{n}, y\_{n}\right)$$

Givens:

x0 = 1

y0 = -1

h = 0.1

 $y^{'}=f\left(x,y\right)=\frac{1}{x^{2}}-\frac{y}{x}-y^{2}$

**Step 0: (1, -1) = (x, y)**

**Step 1: (1.1, -0.9)**

$$x\_{1}=x\_{0}+h=1+0.1=1.1$$

$$f\left(x\_{0}, y\_{0}\right)=\frac{1}{(x\_{0})^{2}}-\frac{y\_{0}}{x\_{0}}-\left(y\_{0}\right)^{2}$$

$$=f\left(1,-1\right)=\frac{1}{1^{2}}-\frac{-1}{1}-\left(-1\right)^{2}=1+1-1=1$$

$$y\_{1}=y\_{0}+hf\left(x\_{0},y\_{0}\right)$$

$$y\_{1}=-1+\left(0.1\right)\left(1\right)=-0.9$$

This eliminates solutions A and D.

For A, the y value is not high enough. Also, the point (1, -1) is not on the graph.

For D, x0 should be at the bottom left corner (1,-1). It is too high.

**Step 2: (1.2, -0.8240)**

$$x\_{2}=x\_{1}+h=1.1+0.1=1.2$$

$$f\left(x\_{1}, y\_{1}\right)=\frac{1}{(x\_{1})^{2}}-\frac{y\_{1}}{x\_{1}}-\left(y\_{1}\right)^{2}$$

$$=f\left(1.1,-0.9\right)=\frac{1}{(1.1)^{2}}-\frac{-0.9}{1.1}-\left(-0.9\right)^{2}=0.7602$$

$$y\_{2}=y\_{1}+hf\left(x\_{1},y\_{1}\right)$$

$$y\_{2}=-0.9+\left(0.1\right)\left(0.7602\right)=-0.8240$$

**Step 3: (1.3, -0.7538)**

$$x\_{3}=x\_{2}+h=1.2+0.1=1.3$$

$$f\left(x\_{2}, y\_{2}\right)=\frac{1}{(x\_{2})^{2}}-\frac{y\_{2}}{x\_{2}}-\left(y\_{2}\right)^{2}$$

$$=f\left(1.2,-0.8240\right)=\frac{1}{(1.2)^{2}}-\frac{-0.8240}{1.2}-\left(-0.8240\right)^{2}=0.7021$$

$$y\_{3}=y\_{2}+hf\left(x\_{2},y\_{2}\right)$$

$$y\_{3}=-0.8240+\left(0.1\right)\left(0.7021\right)=-0.7538$$

**Step 4: (1.4, -0.6935)**

$$x\_{4}=x\_{3}+h=1.3+0.1=1.4$$

$$f\left(x\_{3}, y\_{3}\right)=\frac{1}{(x\_{3})^{2}}-\frac{y\_{3}}{x\_{3}}-\left(y\_{3}\right)^{2}$$

$$=f\left(1.3,-0.7538\right)=\frac{1}{(1.3)^{2}}-\frac{-0.7538}{1.3}-\left(-0.7538\right)^{2}=0.6033$$

$$y\_{4}=y\_{3}+hf\left(x\_{3},y\_{3}\right)$$

$$y\_{4}=-0.7538+\left(0.1\right)\left(0.6033\right)=-0.6935$$

… Steps 5 – 9 …

**Step 9: (2.0, ?)**

From the graph below, the approximation points are ABOVE the graph y = -1/x.

Since with B the approximation points are below the actual graph, the solution must be C.

**Answer: C**

**Graph:**



