Harold's Finances
Cheat Sheet
29 September 2024

Variables

Name	Variable Descriptions
Variables	PV = Original amount, principle, or present value FV = Amount after time t, future value, or face value r = Annual interest rate, rate of growth or loss $(15\% = 0.15)$ k = Number of periods or times per year (quarterly is $k = 4$) t = Number of years n = Number of periods or compoundings $(n = kt)$ i = Effective interest rate per period $\left(i = \frac{r}{k}\right)$ x = Number of payment already made e = Euler's number (~2.71828 18284 59045 …) $PMT = R$ = Equal regular payments towards a loan or Equal periodic payments from an annuity $BAL = B$ = Remaining balance on a loan or annuity DIS = Discount on a U.S. Treasury bill (T − bill) APY = Annual Percentage Yield (APY) or Effective Interest Rate

One-Time Investments

Simple Interest	Discrete	Continuous
Simple Interest	$I = PVrt$	NA
Future Value	$FV = PV + I$ $FV = PV + PVrt$ $FV = PV(1 + rt)$	
Present Value	$PV = \frac{FV}{(1 + rt)}$ $PV = FV(1 + rt)^{-1}$	
T-Bill	$DIS = FVrt$ $\text{Price} = FV - DIS = FV(1 - rt)$ $\text{Effective Rate} = \frac{DIS}{PVt} \bullet 100\%$	
Compounded Interest	**Discrete**	**Continuous**
Compounded Interest	$I = FV - PV$ $I = PV((1 + r)^t - 1)$	$I = PV(e^{rt} - 1)$
Future Value	$FV = PV\left(1 + \frac{r}{k}\right)^{kt}$ If $k = 1$ (annually) then $FV = PV(1 + r)^t$	$FV = PVe^{rt}$

Present Value	$PV = \frac{FV}{\left(1+\frac{r}{k}\right)^{kt}} = FV\left(1+\frac{r}{k}\right)^{-kt}$ If $k = 1$ (annually) then $PV = \frac{FV}{(1+r)^{t}} = FV\,(1+r)^{-t}$	$PV = \frac{FV}{e^{rt}}$ $PV = FVe^{-rt}$
Annual Interest Rate	$r = k\left[\left(\frac{FV}{PV}\right)^{-kt} - 1\right] \bullet 100\ \%$	$r = \frac{1}{t} ln\left(\frac{FV}{PV}\right)$
Annual Percentage Yield (APY) or Effective Interest Rate	$\%\ APY = \left[\left(1+\frac{r}{k}\right)^{kt} - 1\right] \bullet 100\ \%$ $APY = r_E = \left(1+\frac{r}{k}\right)^{k} - 1$	$i = ln\left(\frac{FV}{PV}\right)$

Regular Payments

Compounded Interest	Future Value	Present Value
Number of Periods or Compoundings	$n = kt$	
Effective Interest Rate Per Period	$i = \frac{r}{k}$	
Cost of Loan (Amount You Paid)	$Total_{Paid} = ktPMT$	
Interest You Paid	$I_{Paid} = ktPMT - PV$	
Value of an Ordinary Annuity (PMT at end of period)	$FV = PMT\left[\frac{\left(\left(1+\left(\frac{r}{k}\right)\right)^{kt} - 1\right)}{\left(\frac{r}{k}\right)}\right]$	$PV = PMT\left[\frac{\left(1-\left(1+\left(\frac{r}{k}\right)\right)^{-kt}\right)}{\left(\frac{r}{k}\right)}\right]$
	$FV = PMT\left[\frac{((1+i)^{n} - 1)}{i}\right]$	$PV = PMT\left[\frac{(1-(1+i)^{-n})}{i}\right]$
Value of an Annuity Due (PMT at beginning of period)	$FV = PMT\left[\frac{\left(\left(1+\left(\frac{r}{k}\right)\right)^{kt+1} - 1\right)}{\left(\frac{r}{k}\right)}\right] - PMT$	$PV = PMT + PMT\left[\frac{\left(1-\left(1+\left(\frac{r}{k}\right)\right)^{-kt+1}\right)}{\left(\frac{r}{k}\right)}\right]$
	$FV = PMT\left[\frac{((1+i)^{n+1} - 1)}{i}\right] - PMT$	$PV = PMT + PMT\left[\frac{(1-(1+i)^{-(n-1)})}{i}\right]$
Amortization Payment Amount	$PMT = FV\left[\frac{\left(\left(1+\left(\frac{r}{k}\right)\right)^{kt} - 1\right)}{\left(\frac{r}{k}\right)}\right]^{-1}$	$PMT = PV\left[\frac{\left(1-\left(1+\left(\frac{r}{k}\right)\right)^{-kt}\right)}{\left(\frac{r}{k}\right)}\right]^{-1}$
	$PMT = FV\left[\frac{i}{((1+i)^{n} - 1)}\right]$	$PMT = PV\left[\frac{i}{(1-(1+i)^{-n})}\right]$
Remaining Balance	NA	$BAL = PMT\left[\frac{\left(1-\left(1+\left(\frac{r}{k}\right)\right)^{-kt+x}\right)}{\left(\frac{r}{k}\right)}\right]$
	NA	$BAL = PMT\left[\frac{(1-(1+i)^{-(n-x)})}{i}\right]$

Examples

Scenario	Calculations
Savings Account: $PV = \$100$ $r = 8\% = 0.08$ $t = 1\ year$	If $k = 0$, $FV = \$100.00\ (+0¢)$ Simple If $k = 1$, $FV = \$108.00\ (+\$8)$ Annually If $k = 2$, $FV = \$108.16\ (+16¢)$ Semiannually If $k = 4$, $FV = \$108.24\ (+8¢)$ Quarterly If $k = 12$, $FV = \$108.30\ (+6¢)$ Monthly If $k = 52$, $FV = \$108.32\ (+2¢)$ Weekly If $k = 365$, $FV = \$108.33\ (+1¢)$ Daily If $k \to \infty$, $FV = \$108.33\ (+0¢)$ Continuously
House Mortgage Payment: $PV = \$300{,}000$ (home loan) $PMT =$ Equal periodic payments $r = 3.5\% = 0.035$ $k = 12$ (monthly) $t = 30$ years	$PMT = PV\left[\frac{\left(\frac{r}{k}\right)}{\left(1-\left(1+\left(\frac{r}{k}\right)\right)^{-kt}\right)}\right]$ $PMT = \$300{,}000\left[\frac{\left(\frac{0.035}{12}\right)}{\left(1-\left(1+\left(\frac{0.035}{12}\right)\right)^{-(12)(30)}\right)}\right]$ $\boldsymbol{PMT = \$1{,}347.13/month}$
Loan Cost Analysis	$t = 30$ years: Cost of loan $= ktPMT = (12)(30)(\$1{,}347.13) = \$484{,}966.80$ Interest paid $= ktPMT - PV = \$484{,}966.80 - \$300{,}000 =$ $184,966.80 $t = 15$ years: Cost of loan $= ktPMT = (12)(15)(\$2{,}144.65) = \$386{,}037.00$ Interest paid $= ktPMT - PV = \$386{,}037.00 - \$300{,}000 =$ $86,037.00
Rule of 72	How long to double your money at a given interest rate r. $FV = 2PV = PVe^{rt}$ $e^{rt} = 2$ $rt = \ln 2 = 0.693 \cong 0.72$ $t \cong \frac{0.72}{r}$ $If\ r = 4\%\ then\ t \cong \frac{72}{4} = 18\ years$ Actual: $t = \frac{\ln 2}{0.04} = 17.325\ years \cong 17\ years\ 4\ months$

