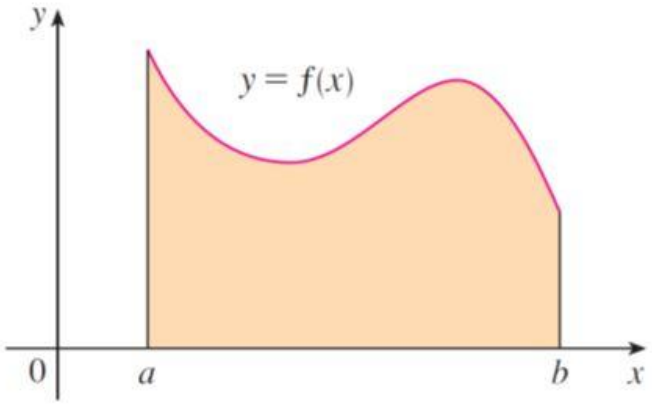
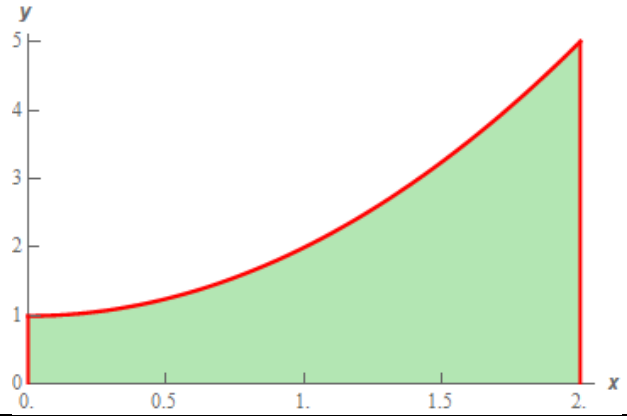
Harold's Fundamental Theorem of Calculus
Cheat Sheet

5 May 2026

The First Fundamental Theorem of Calculus: Integrating Derivatives

Formula	Example
Upper Bound Minus Lower Bound Formula $$\int_a^b f(x)\,dx = \int_a^b F'(x)\,dx = F(x)\Big\|_a^b = F(b) - F(a)$$  	**Solve:** $$\int_0^2 x^2 + 1\,dx$$ 1st FToC Formula $$\int_a^b f(x)\,dx = F(b) - F(a)$$ Identify functions $$f(x) = F'(x) = x^2 + 1$$ $$F(x) = \frac{x^3}{3} + x$$ Plug into formula $$F(b) - F(a)$$ $$= F(2) - F(0)$$ $$= \left[\frac{(2)^3}{3} + (2)\right] - \left[\frac{(0)^3}{3} + 0\right]$$ $$= \frac{14}{3} - 0 = \frac{14}{3} \approx 4.6667$$ 

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The Second Fundamental Theorem of Calculus: Differentiating Integrals

Formula

Simple Formula

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Single Function Formula

$$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f\big(g(x)\big)g'(x)$$

General Formula

$$\boldsymbol{\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f\big(h(x)\big)h'(x) - f\big(g(x)\big)g'(x)}$$

Proof:

a) Apply the First Fundamental Theorem of Calculus

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = \frac{d}{dx}[F(t)]_{g(x)}^{h(x)}$$

$$= \frac{d}{dx}\big[F\big(h(x)\big) - F\big(g(x)\big)\big]$$

b) Distribute

$$= \frac{d}{dx}F\big(h(x)\big) - \frac{d}{dx}F\big(g(x)\big)$$

c) Chain rule

$$= F'\big(h(x)\big)\,\frac{d}{dx}h(x) - F'\big(g(x)\big)\,\frac{d}{dx}g(x)$$

d) Simplify

$$= F'\big(h(x)\big)\,h'(x) - F'\big(g(x)\big)\,g'(x)$$

e) Substitute

$$= f\big(h(x)\big)\,h'(x) - f\big(g(x)\big)\,g'(x)$$

Example

Solve:

$$\frac{d}{dx}\int_{4x}^{x^2} e^t\,dt$$

2nd FToC General Formula

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f\big(h(x)\big)\,h'(x) - f\big(g(x)\big)\,g'(x)$$

Identify functions

$$f(t) = e^t$$
$$g(x) = 4x$$
$$h(x) = x^2$$

Substitute

$$f\big(g(x)\big) = e^{4x}$$
$$f\big(h(x)\big) = e^{x^2}$$

Differentiate

$$g'(x) = \frac{d}{dx}4x = 4$$
$$h'(x) = \frac{d}{dx}x^2 = 2x$$

Plug into formula

$$= f\big(h(x)\big)\,h'(x) - f\big(g(x)\big)\,g'(x)$$
$$= e^{x^2}(2x) - e^{4x}(4)$$
$$= \boldsymbol{2xe^{x^2} - 4e^{4x}}$$

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