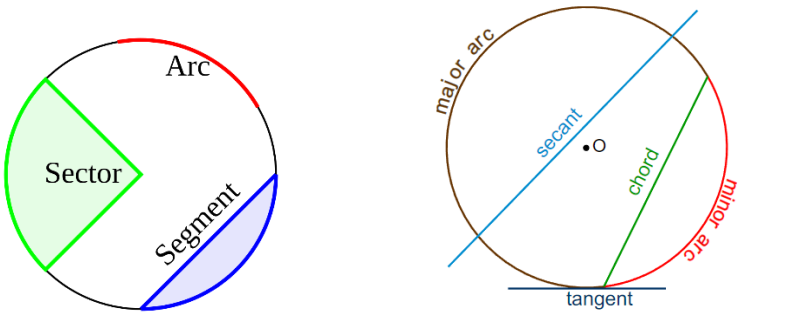
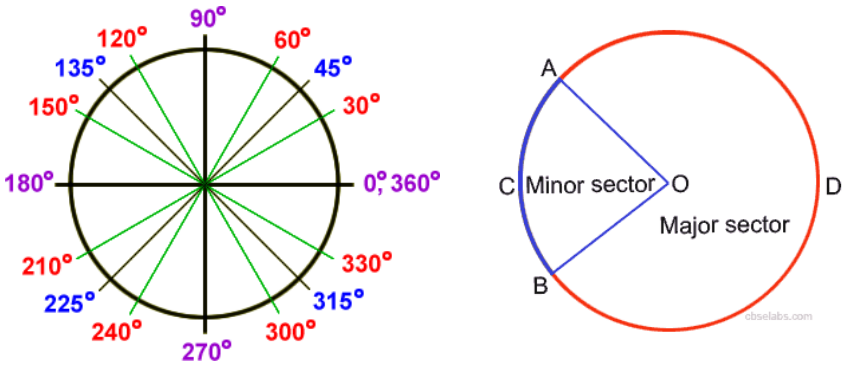
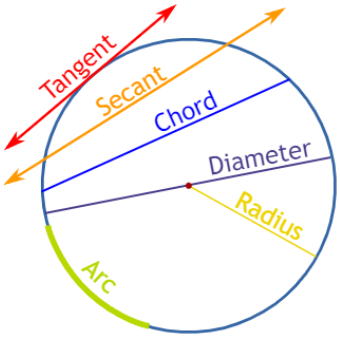
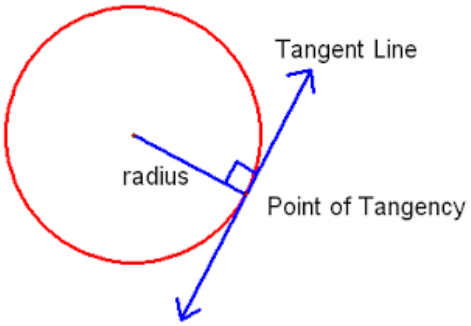
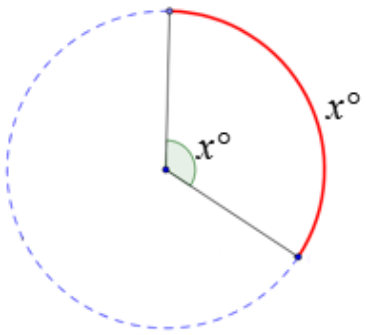
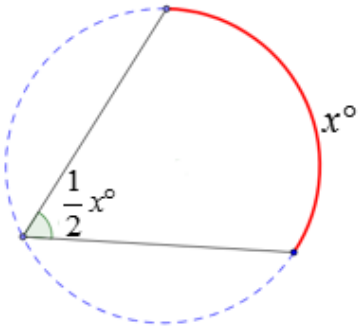
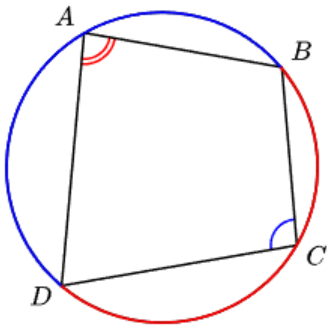
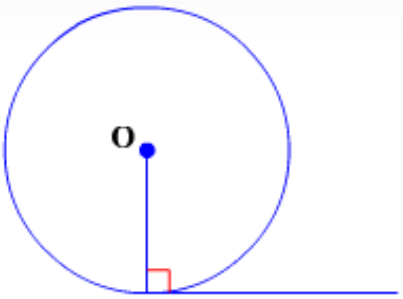
Harold's Geometry – Circle Theorems
Cheat Sheet

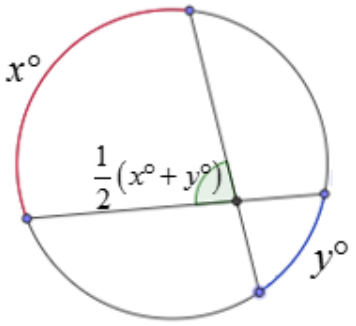
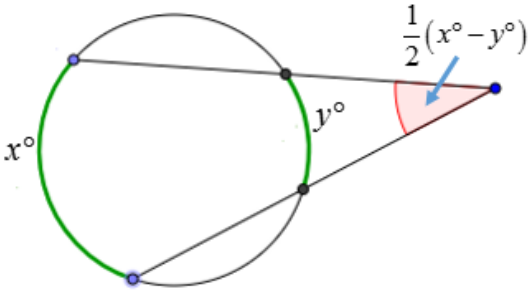
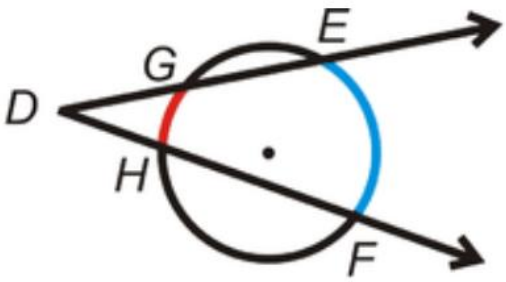
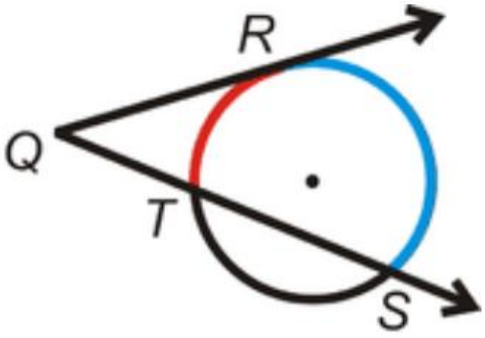
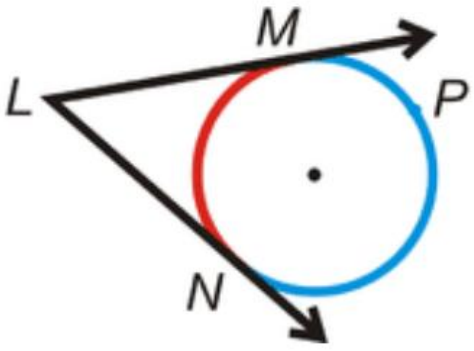
1 September 2025

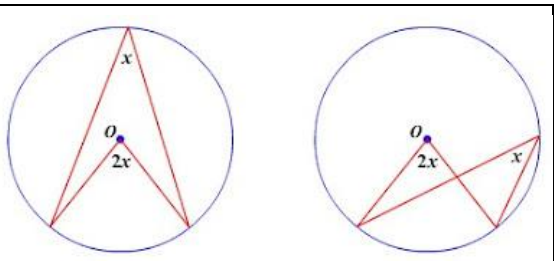
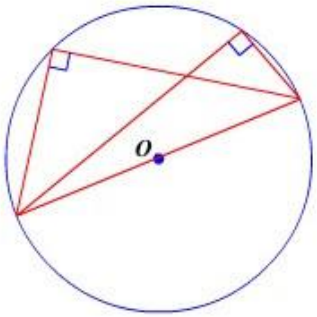
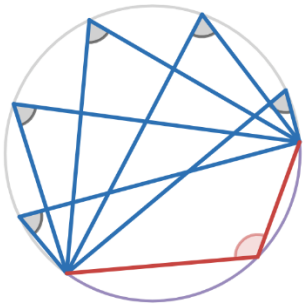
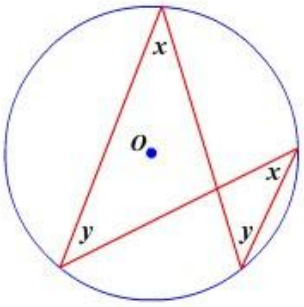
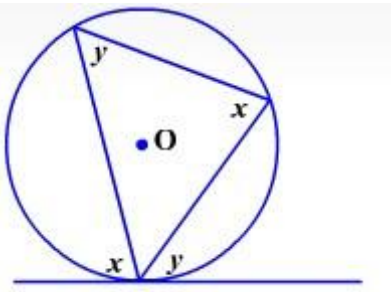
Terminology

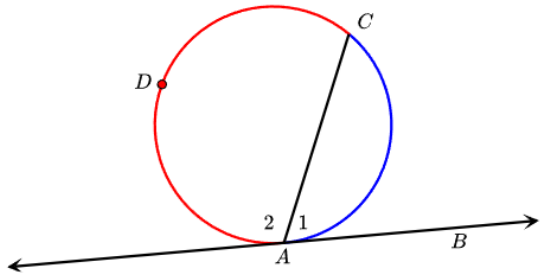
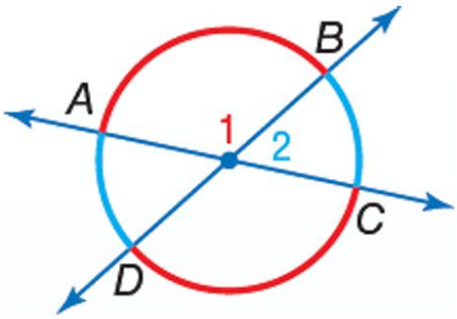
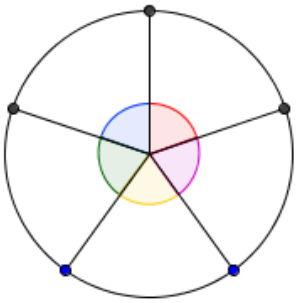
Category	Examples
Arcs	
Angles & Sectors	
Lines & Chords	
Tangents	

Arcs and Angles in a Circle

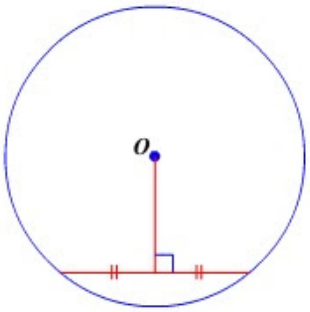
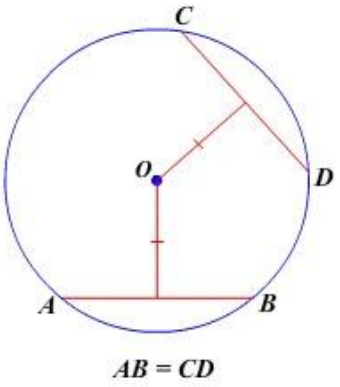
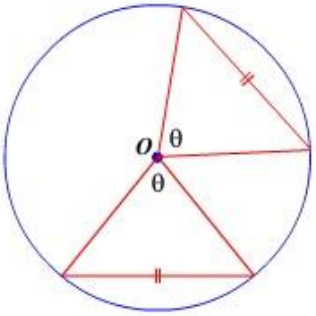
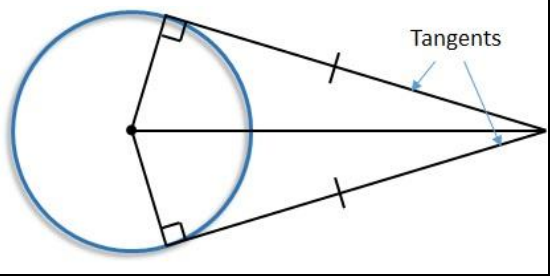
Configuration	Rule / Formula	Diagram
Central Angle (Angle at Center)	Equal to arc $\theta = x^\circ$ $m\angle ABC = m\widehat{AC}$	
Inscribed Angle (Angle in Same Segment)	Half the arc $\theta = \frac{1}{2}x^\circ$	
Inscribed Quadrilateral (Opposite Angles of Cyclic Quadrilateral)	$m\angle A + m\angle C = 180^\circ$ $m\angle B + m\angle D = 180^\circ$ The opposite angles of cyclic quadrilaterals are supplementary (180°).	
Radius ⊥ Tangent	The angle between the radius and a tangent is 90°.	

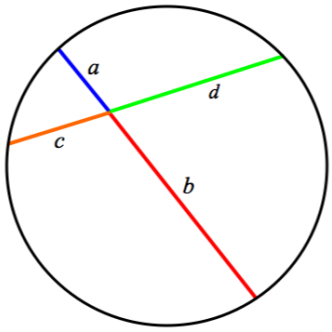
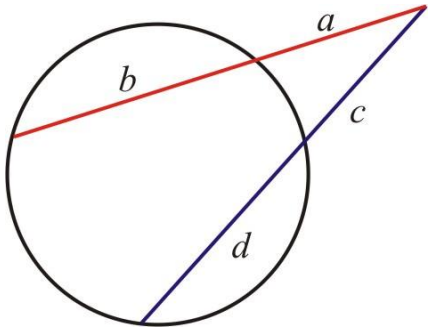
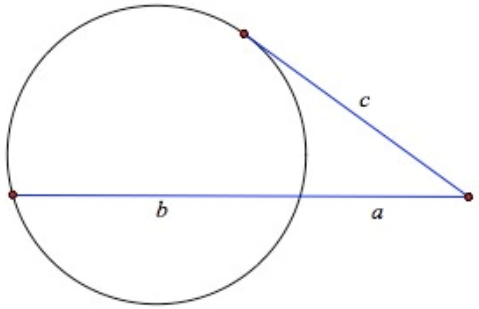
Two Chords (Internal Angle)	Half the sum $\theta = \frac{1}{2}(x^\circ + y^\circ)$	
Two Secants (External Angle)	Half the difference $\theta = \frac{1}{2}(x^\circ - y^\circ)$ $m\angle D = \frac{1}{2}(m\widehat{EF} - m\widehat{GH})$	 
Secant & Tangent (External Angle)	$m\angle Q = \frac{1}{2}(m\widehat{RS} - m\widehat{RT})$	
Two Tangents (External Angle)	$m\angle L = \frac{1}{2}(m\widehat{MPN} - m\widehat{MN})$	

Angle at Center	$2x^{\circ} vs. x^{\circ}$ The angle at the center is twice the angle standing on the same chord/arc.	
Angles Inscribed in a Semi-Circle	Right Angles (90°) Angles on a semi-circle are 90°.	
Angles Inscribed in a Circle	Angles from two points on a circle are equal.	
Same Segment Theorem (Two Inscribed Angles)	$x^{\circ} = x^{\circ}$ $y^{\circ} = y^{\circ}$ Angles on the same arc are equal.	
Alternate Segment Theorem	$x^{\circ} = x^{\circ}$ $y^{\circ} = y^{\circ}$ The angle between a chord and a tangent is equal to the angle in the alternate segment.	

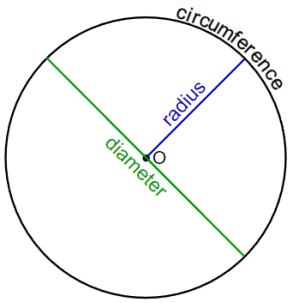
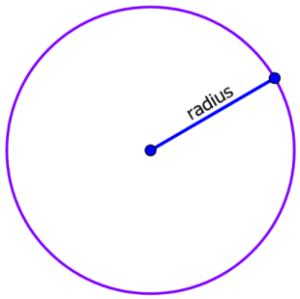
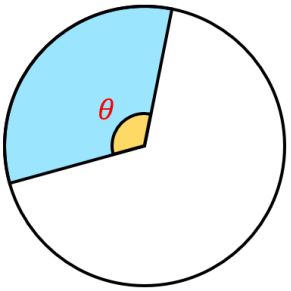
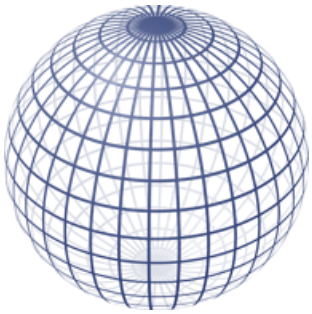
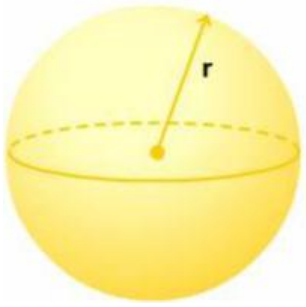
Tangent and Intersected Chord Theorem	$m\angle 1 = \frac{1}{2}(m\widehat{AC})$ $m\angle 2 = \frac{1}{2}(m\widehat{ADC})$ If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.	
Supplimentary Angles	$m\angle 1 + m\angle 2 = 180°$	
Interior Angles	$\theta = \frac{360°}{n}$ Sum of interior angles of a circle is always 360°.	

Chords and Secants in a Circle

Configuration	Rule / Formula	Diagram
Perpendicular Bisector of Chord Passes Through Center	The line from the center of a circle to the center of a chord is perpendicular to the chord. A perpendicular line from the chord to the center bisects the chord.	
Equal Chords Equidistant from Center	Equal chords are equal distance from the center. Chords that are equal distance from the center are equal.	 $AB = CD$
Equal Arcs, Equal Chords	Equal arc/chord subtend equal angles at the center. Equal angles stand on an equal arc/chord.	
Tangents from External Point	Tangent segments drawn from an external point are equal.	

Intersecting Chords Theorem	$a \cdot b = c \cdot d$	
Intersecting Secants Theorem	$a \cdot (a + b) = c \cdot (c + d)$	
Intersecting Secant-Tangent Theorem	$a(a + b) = c^2$	

Area and Perimeter

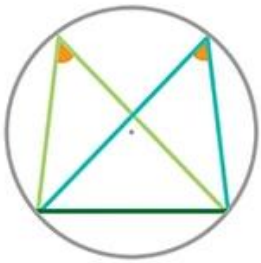
Configuration	Rule / Formula	Diagram
Radius	r The distance from the center or origin to a point on the circle.	
Diameter	$d = 2r$	
Circumference	$C = 2\pi r$ $C = \pi d$	
Area of Circle	$A = \pi r^2$	
Area of a Sector	$A = \left(\frac{\theta^\circ}{360^\circ}\right) \cdot \pi r^2$ $where \left(\frac{\theta^\circ}{360^\circ}\right) = \frac{area\ of\ sector}{area\ of\ circle}$	
Surface Area of Sphere	$SA = 4\pi r^2$	
Volume of Sphere	$V = \frac{4}{3}\pi r^3$	

CIRCLE THEOREMS

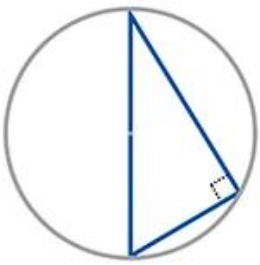
TIGER MOON

Circles have a number of different angle properties, these are described as Theorems.

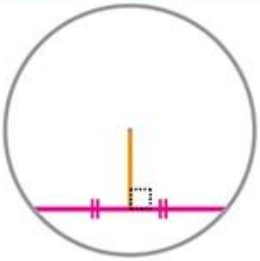
Angles in the same segment and standing on the same chord are always equal.



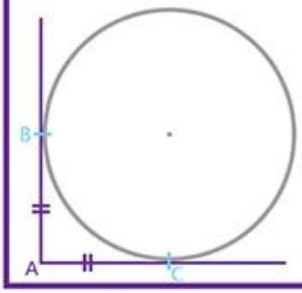
The angle in a semi-circle is always 90°.



A line drawn from the centre of a circle to the mid-point of a chord is perpendicular to the chord at 90°.

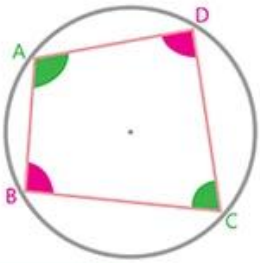


Tangents from a common point (A) to a circle are always equal in length.
AB = AC

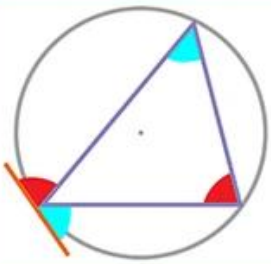


The angle at the centre of a circle is twice the angle at the circumference.

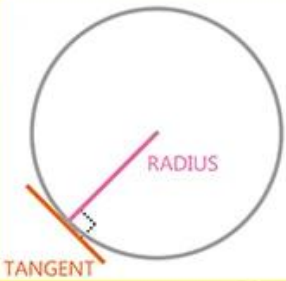
ABCD is a cyclic quadrilateral, all vertices lie on the circumference of the circle. Opposite angles add up to 180°.
A+C=180° B+D=180°



The angle between the tangent and the side of the triangle is equal to the interior opposite angle.



The angle between the tangent and the radius is always 90°.



KEY WORDS:
- SUBTENDED
- CIRCUMFERENCE
- TANGENT
- PERPENDICULAR
- OPPOSITE
- ANGLE
- CHORD
- SEGMENT

Sources

- Kevin's Online Maths, Rules of Circle Geometry, http://kelvinsonlinemaths.blogspot.com/2011/03/rules-of-circle-geometry.html
- Geometry R, Unit 13 – Circles, Mr. Ross @ Grady High, https://mrrossatgradyhigh.files.wordpress.com/2022/08/unit-13-notes-circles_2018.pdf
- Pinterest, Tangent & Secant Lines, Sandy Lakey, https://www.pinterest.com.mx/pin/817403401103649163/
- Online Math Learning.com, Angles and Intercepted Arcs, https://www.onlinemathlearning.com/arc-angles.html
    - ck-12, 9.7 Segments of Secants and Tangents, https://www.ck12.org/book/ck-12-foundation-and-leadership-public-schools-college-access-reader%3a-geometry/section/9.7/
    - ck-12, Angles Outside a Circle, https://www.ck12.org/c/geometry/angles-outside-a-circle/lesson/Angles-Outside-a-Circle-BSC-GEOM/
- Tiger Moon (2025). GCSE Maths Circle Theorems - A2 poster. https://www.tigermoon.co.uk/collections/maths-gcse-posters/products/circle-theorems-maths-poster