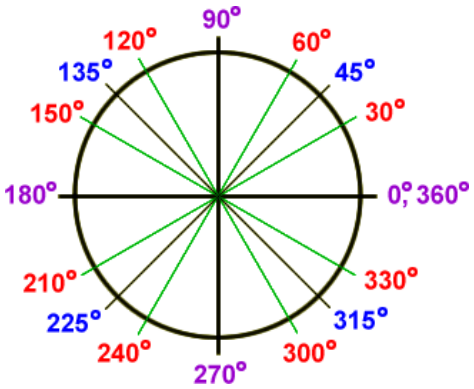
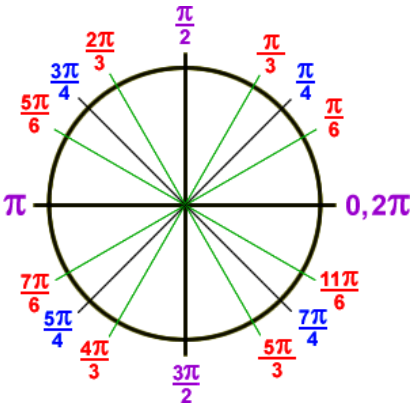
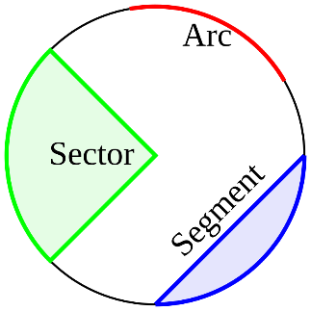
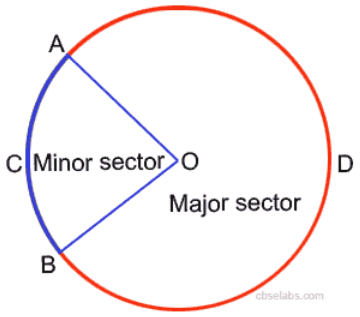
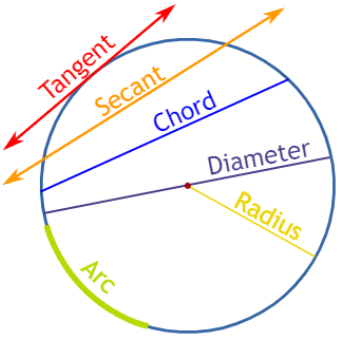
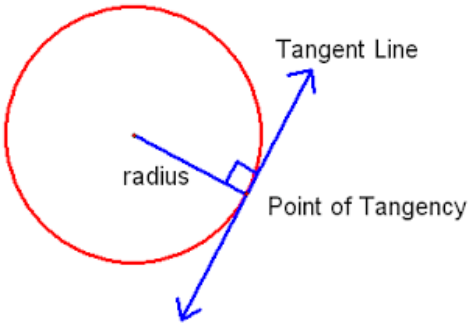
Harold's Geometry – Circle Theorems
Cheat Sheet

6 August 2026

Terminology

Category	Examples
Angles	 Degrees  Radians
Arcs & Sectors	 
Lines, Chords & Tangents	 

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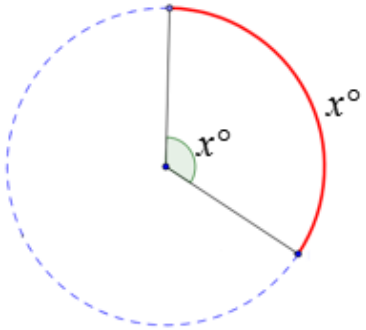
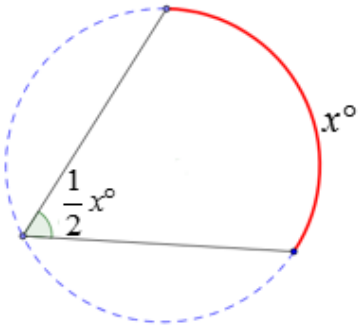
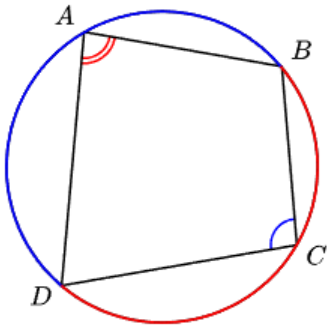
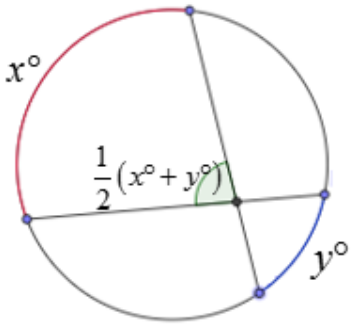
Internal Arcs and Angles

Configuration	Rule / Formula	Diagram
Central Angle (Angle at Center)	Equal to arc $\theta = x^\circ$ $m\angle AOC = m\widehat{AC}$	
Inscribed Angle (Angle in Same Segment)	Half the arc $\theta = \frac{1}{2}x^\circ$	
Inscribed Quadrilateral (Opposite Angles of Cyclic Quadrilateral)	$m\angle A + m\angle C = 180^\circ$ $m\angle B + m\angle D = 180^\circ$ The opposite angles of cyclic quadrilaterals are supplementary (180°).	
Two Chords (Internal Angle)	Half the sum $\theta = \frac{1}{2}(x^\circ + y^\circ)$	

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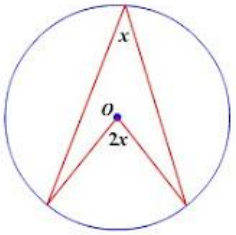
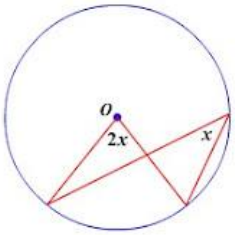
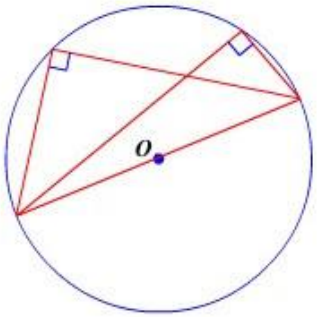
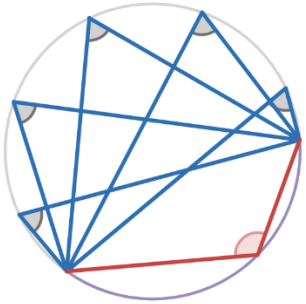
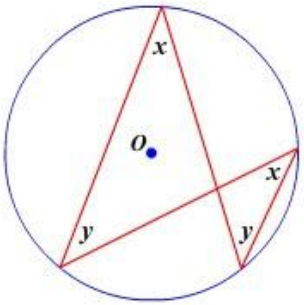
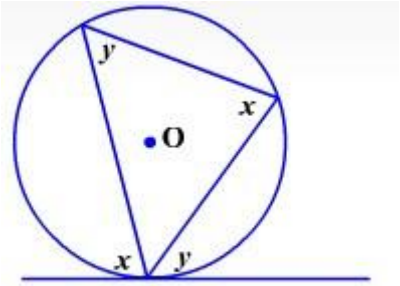
Angle at Center	$2x^{\circ} vs. x^{\circ}$ The angle at the center is twice the angle standing on the same chord/arc.	 Symmetrical  Asymmetrical
Angles Inscribed in a Semi-Circle	Right Angles (90°) An angle inscribed in a semicircle is a right angle (90°).	
Angles Inscribed in a Circle	Inscribed angles that intercept the same arc are congruent.	
Same Segment Theorem (Two Inscribed Angles)	$x^{\circ} = x^{\circ}$ $y^{\circ} = y^{\circ}$ Angles subtended by the same chord in the same segment are equal.	
Alternate Segment Theorem	$x^{\circ} = x^{\circ}$ $y^{\circ} = y^{\circ}$ The angle between a chord and a tangent is equal to the angle in the alternate segment.	

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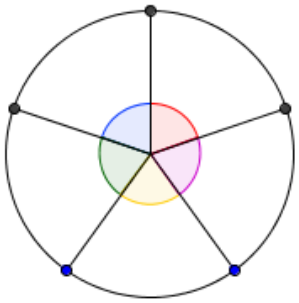
Angles at the Center	$\theta = \frac{360°}{n}$ The sum of the central angles of a circle is always 360°.	

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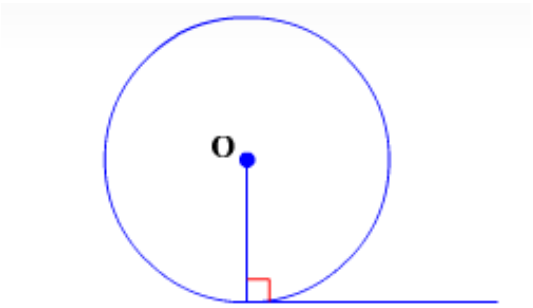
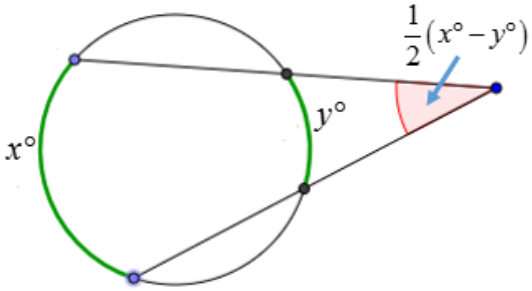
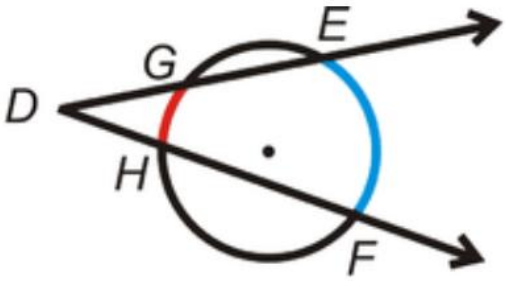
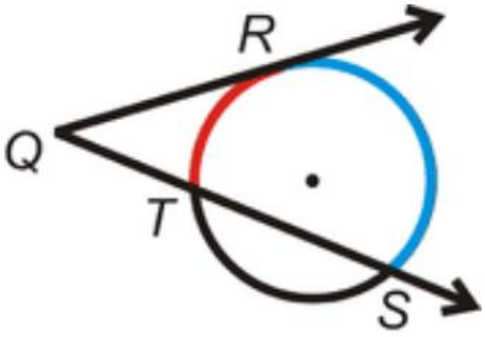
External Tangents and Secants

Configuration	Rule / Formula	Diagram
Radius ⊥ Tangent	The radius drawn to the point of tangency is perpendicular (90°) to the tangent.	
Two Secants (External Angle)	Half the difference $\theta = \frac{1}{2}(x^\circ - y^\circ)$ $m\angle D = \frac{1}{2}(m\widehat{EF} - m\widehat{GH})$	 
Secant & Tangent (External Angle)	$m\angle Q = \frac{1}{2}(m\widehat{RS} - m\widehat{RT})$	

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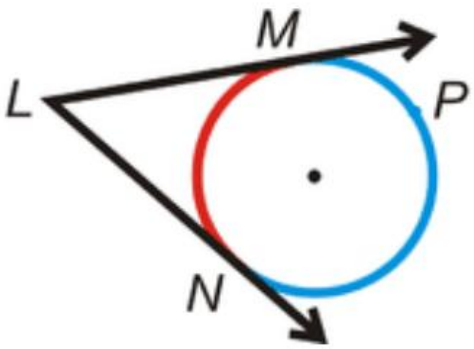
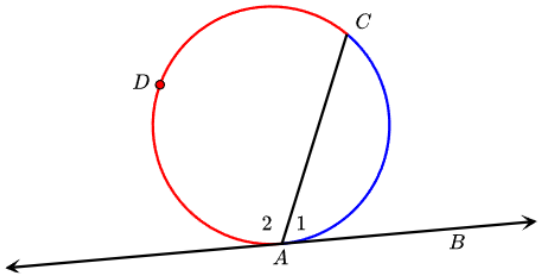
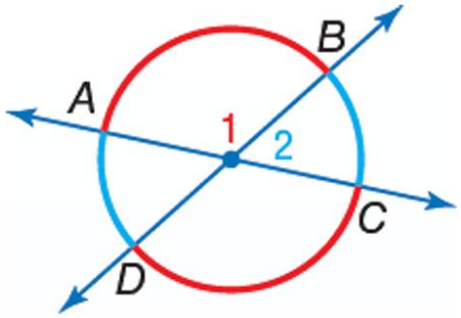
Two Tangents (External Angle)	$m\angle L = \frac{1}{2}\left(m\widehat{MPN} - m\widehat{MN}\right)$	
Tangent and Intersected Chord Theorem	$m\angle 1 = \frac{1}{2}\left(m\widehat{AC}\right)$ $m\angle 2 = \frac{1}{2}\left(m\widehat{ADC}\right)$ If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.	
Supplementary Angles	$m\angle 1 + m\angle 2 = 180°$ Adjacent angles that form a linear pair are supplementary.	

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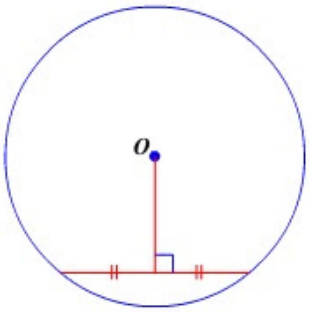
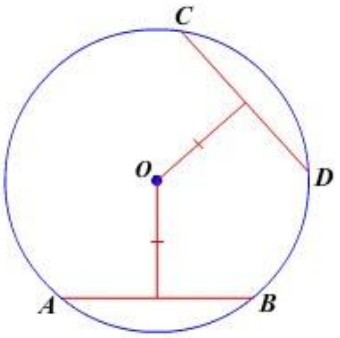
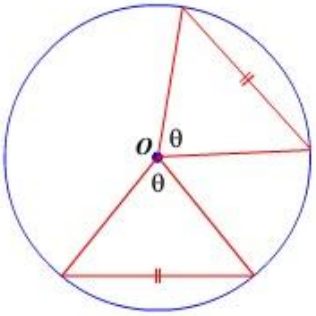
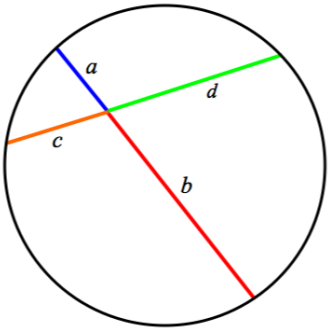
Internal Chords

Configuration	Rule / Formula	Diagram
Perpendicular Bisector of Chord Passes Through Center	The line from the center of a circle to the midpoint of a chord is perpendicular to the chord. A perpendicular line from the chord to the center bisects the chord.	
Equal Chords Equidistant from Center	Equal chords are equal perpendicular distances from the center. Chords that are equidistant from the center are equal in length.	 $AB = CD$
Equal Arcs, Equal Chords	Equal chords correspond to equal arcs and equal central angles.	
Euclid's Intersecting Chords Theorem	$a \cdot b = c \cdot d$ The intersecting angles do not have to be right angles.	

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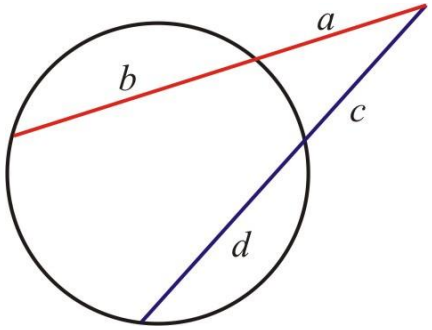
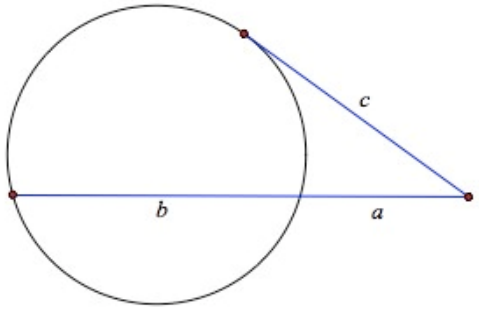
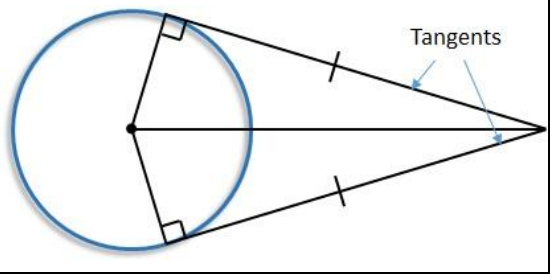
External Tangents and Secants

Configuration	Rule / Formula	Diagram
Intersecting Secants Theorem	$a \cdot (a + b) = c \cdot (c + d)$	
Intersecting Secant-Tangent Theorem	$a(a + b) = c^2$	
Tangents from External Point	Tangent segments drawn from an external point are equal.	

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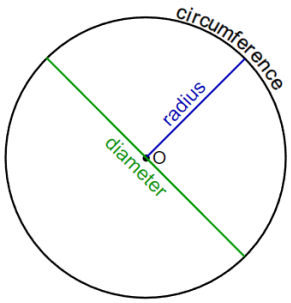
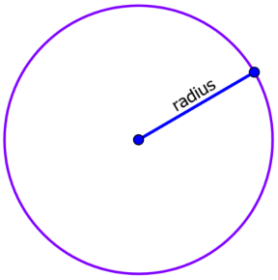
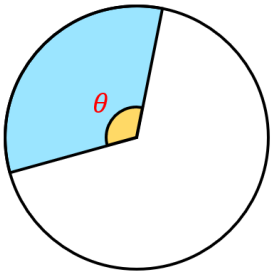
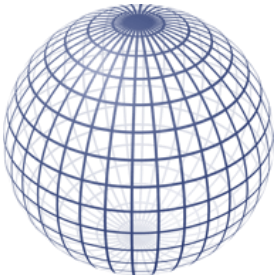
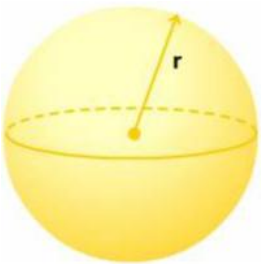
Area and Perimeter

Configuration	Rule / Formula	Diagram
Radius	r	
Diameter	$d = 2r$	
Circumference	$C = 2\pi r = \pi d$	
Arc Length	$s = r\theta$ *where θ is in radians*	$s = \frac{\theta}{360°}(2\pi r)$ *where θ is in degrees*
Area of Circle	$A = \pi r^2$	
Area of a Sector	$A = \frac{1}{2}r^2\theta$ $A = \left(\frac{\theta^\circ}{360^\circ}\right) \cdot \pi r^2$ $where \left(\frac{\theta^\circ}{360^\circ}\right) = \frac{\text{A}_{\text{sector}}}{\text{A}_{\text{circle}}}$	
Surface Area of Sphere	$SA = 4\pi r^2$	
Volume of Sphere	$V = \frac{4}{3}\pi r^3$	

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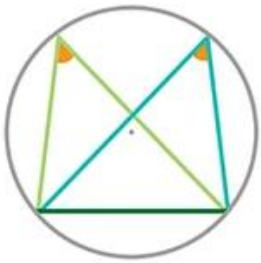
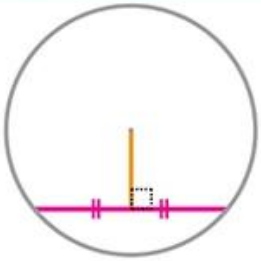
CIRCLE THEOREMS

TIGER MOON

Angles in the same segment and standing on the same chord are always equal.

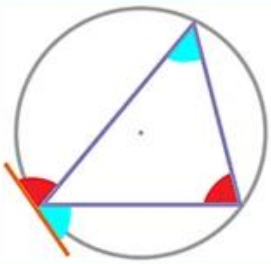


A line drawn from the centre of a circle to the mid-point of a chord is perpendicular to the chord at 90°.



The angle at the centre of a circle is twice the angle at the circumference.

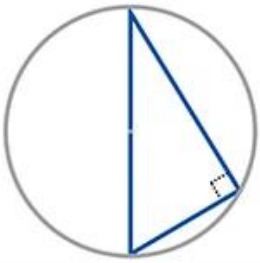
The angle between the tangent and the side of the triangle is equal to the interior opposite angle.



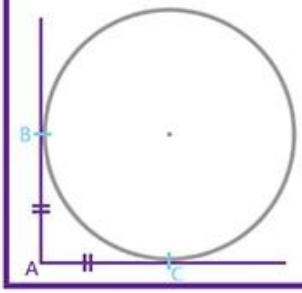
KEY WORDS:

- SUBTENDED
- CIRCUMFERENCE
- TANGENT
- PERPENDICULAR
- -OPPOSITE
- -ANGLE
- CHORD
- SEGMENT

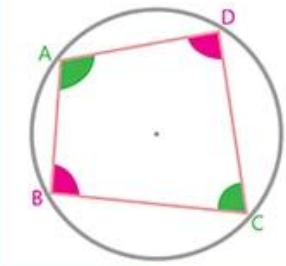
Circles have a number of different angle properties, these are described as Theorems.



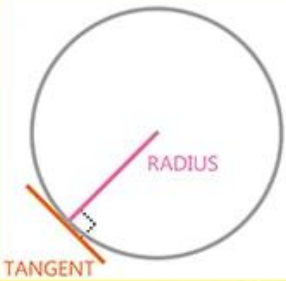
The angle in a semi-circle is always 90°.



Tangents from a common point (A) to a circle are always equal in length.
AB = AC



ABCD is a cyclic quadrilateral, all vertices lie on the circumference of the circle. Opposite angles add up to 180°.
A+C=180° B+D=180°



The angle between the tangent and the radius is always 90°.

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Sources

- Kevin's Online Maths, Rules of Circle Geometry, http://kelvinsonlinemaths.blogspot.com/2011/03/rules-of-circle-geometry.html (inactive source)
- Geometry R, Unit 13 – Circles, Mr. Ross @ Grady High, https://mrrossatgradyhigh.files.wordpress.com/2022/08/unit-13-notes-circles_2018.pdf
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    - ck-12, Angles Outside a Circle, https://www.ck12.org/c/geometry/angles-outside-a-circle/lesson/Angles-Outside-a-Circle-BSC-GEOM/
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