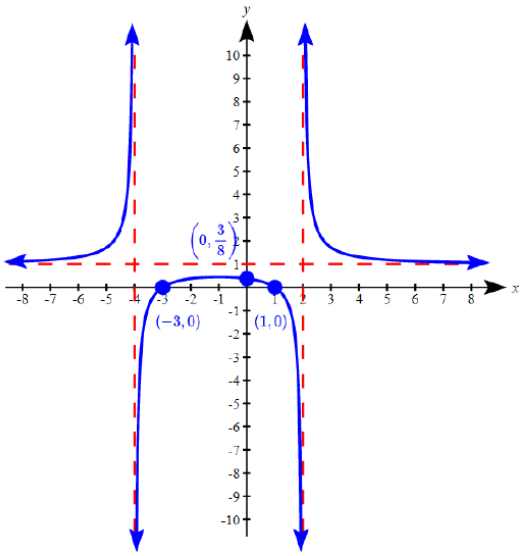
Harold's Graphing Rationals
Cheat Sheet

5 May 2026

Graphing Rational Functions	How to Obtain	
$f(x) = \frac{P(x)}{Q(x)} = \frac{ax^n + \cdots + b}{cx^m + \cdots + d}$	Reorder from highest to lowest degree/power.	
Horizontal Asymptote (HA) EBA = Endpoint Behavior Analysis BOTNO = **B**igger **O**n **T**op, **NO** HA BOSCO = **BO**th are **S**ame, take **CO**efficients BOBO = **B**igger **O**n **B**ottom, HA is $y = \mathbf{0}$	**Left:**	
	Case 1: $n > m$ (BOTNO)	$\lim_{x\to\infty} \frac{ax^n}{cx^m} \to \infty$
	Case 2: $n = m$ (BOSCO)	$\lim_{x\to\infty} \frac{ax^n}{cx^m} = \frac{a}{c}$ (line)
	Case 3: $n < m$ (BOBO)	$\lim_{x\to\infty} \frac{ax^n}{cx^m} = 0$
Slant Asymptote (SA)	Case 4: $n = m + 1$ (Special case of BOTNO)	$\frac{ax^n}{cx^m} \to \frac{a}{c}x + k$
	Use synthetic or long division to determine k.	
Holes	Cancel identical factors in numerator and denominator.	
y-Intercept	**Right:** Plug in $x = 0$ to get $y = \frac{b}{d}$.	
x-Intercepts (roots or zeros)	**Top:** Factor to find roots of $P(x)$, check for holes.	
Vertical Asymptotes (VA)	**Bottom:** Factor to find roots of $Q(x)$, check for holes.	
Domain (valid x values)	All x except for VAs and holes.	
Range (valid y values)	Depends upon Domain.	
Examples		

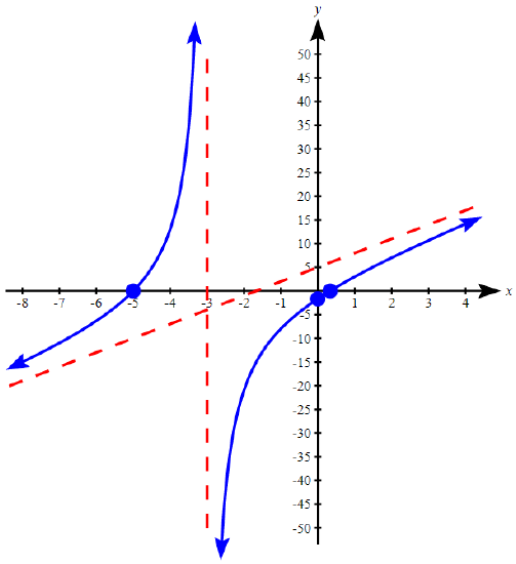
Left Case 2: $n = m$

$$f(x) = \frac{x^2 + 2x - 3}{x^2 + 2x - 8} = \frac{(x+3)(x-1)}{(x-2)(x+4)}$$



Left Case 4: $n = m + 1$

$$f(x) = \frac{3x^2 + 14x - 5}{x + 3} = \frac{(x+5)(3x-1)}{x+3}$$



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