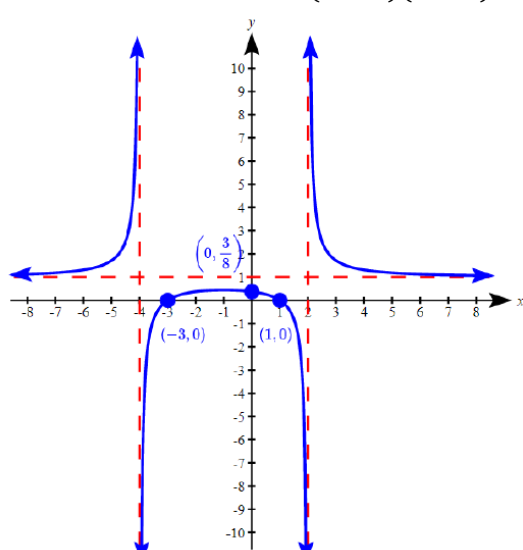


Harold's Graphing Rationals

Cheat Sheet

31 March 2025

Graphing Rational Functions	How to Obtain	
$f(x) = \frac{P(x)}{Q(x)} = \frac{\boxed{ax^n + \dots + b}}{\boxed{cx^m + \dots + d}}$	Reorder from highest to lowest degree/power.	
Horizontal Asymptote (HA) EBA = Endpoint Behavior Analysis BOTNO = B igger O n T op, N O HA BOSCO = B Oth are S ame, take C Oefficients BOBO = B igger O n B ottom, HA is $y = 0$	Left:	
	Case 1: $n > m$ (BOTNO)	$\lim_{x \rightarrow \infty} \frac{ax^n}{cx^m} \rightarrow \infty$
	Case 2: $n = m$ (BOSCO)	$\lim_{x \rightarrow \infty} \frac{ax^n}{cx^m} = \frac{a}{c}$ (line)
	Case 3: $n < m$ (BOBO)	$\lim_{x \rightarrow \infty} \frac{ax^n}{cx^m} = 0$
Slant Asymptote (SA)	Case 4: $n = m + 1$ (Special case of BOTNO)	$\frac{ax^n}{cx^m} \rightarrow \frac{a}{c}x + k$
	Use synthetic or long division to determine k .	
Holes	Cancel identical factors in numerator and denominator.	
y-Intercept	Right: Plug in $x = 0$ to get $y = \frac{b}{d}$.	
x-Intercepts (roots or zeros)	Top: Factor to find roots of $P(x)$, check for holes.	
Vertical Asymptotes (VA)	Bottom: Factor to find roots of $Q(x)$, check for holes.	
Domain (valid x values)	All x except for VAs and holes.	
Range (valid y values)	Depends upon Domain.	
Examples		
Left Case 2: $n = m$ $f(x) = \frac{x^2 + 2x - 3}{x^2 + 2x - 8} = \frac{(x + 3)(x - 1)}{(x - 2)(x + 4)}$ Left Case 4: $n = m + 1$ $f(x) = \frac{3x^2 + 14x - 5}{x + 3} = \frac{(x + 5)(3x - 1)}{x + 3}$