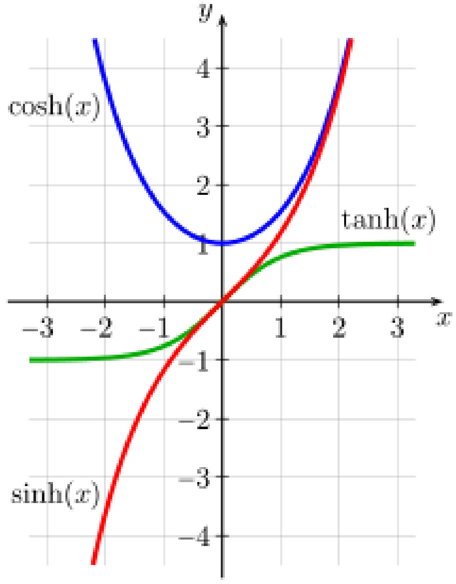
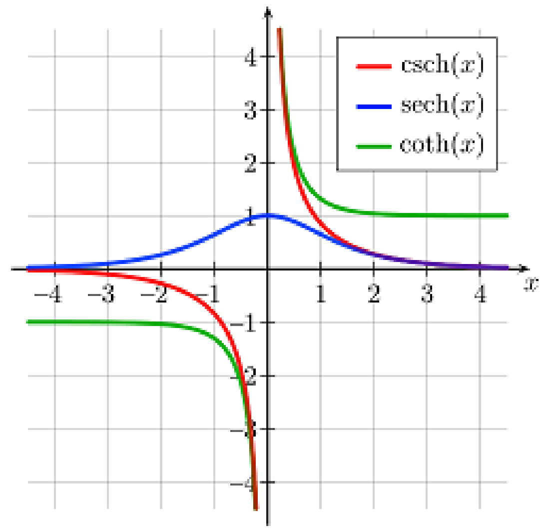
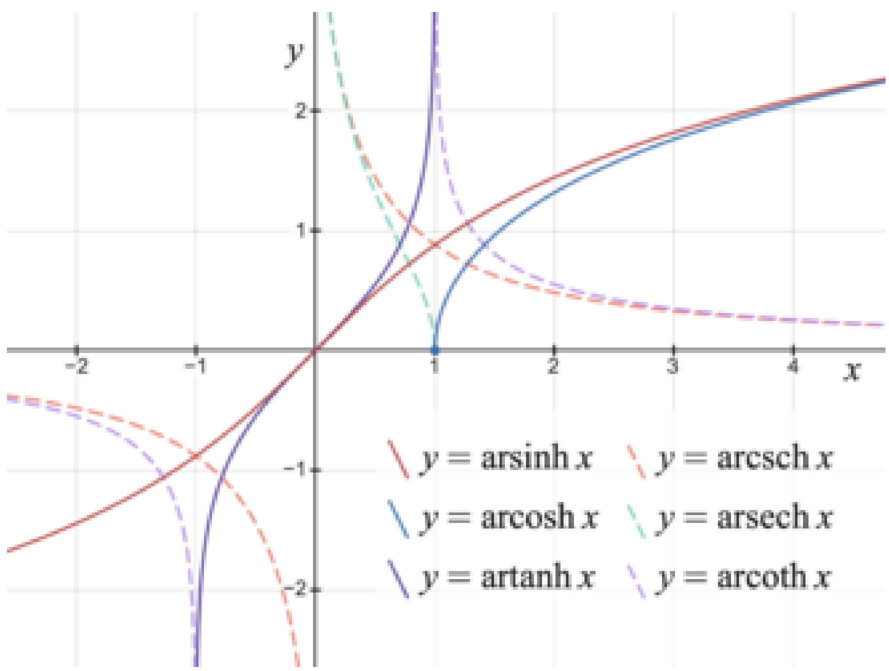
Harold's Hyperbolics
Cheat Sheet
22 September 2025

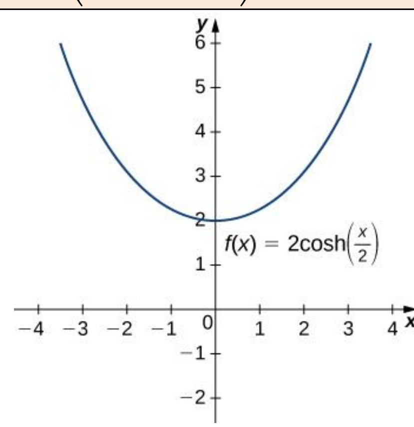
Hyperbolic Graphs



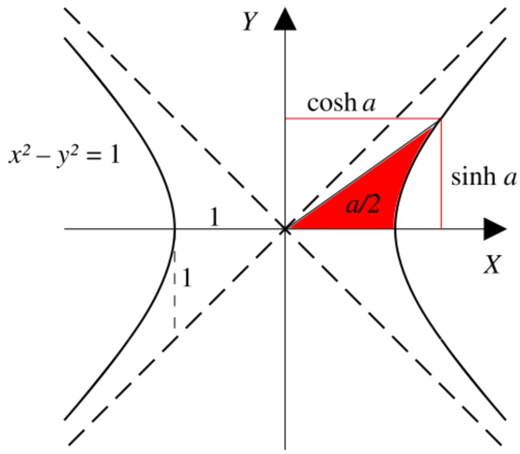


Inverse Hyperbolic Graphs



Hyperbolic Definitions		$sinh$ & $cosh$	e^x	Complex
1. Hyperbolic Sine	$sinh(x)$	$\frac{1}{cosh(x)}$	$\frac{e^x - e^{-x}}{2}$	$-i\,sin(ix)$
2. Hyperbolic Cosine (catenary)	$cosh(x)$	$\frac{1}{sinh(x)}$	$\frac{e^x + e^{-x}}{2}$	$cos(ix)$
3. Hyperbolic Tangent	$tanh(x)$	$\frac{sinh(x)}{cosh(x)}$	$\frac{e^x - e^{-x}}{e^x + e^{-x}}$	$-i\,tan(ix)$
4. Hyperbolic Cotangent	$coth(x)$	$\frac{cosh(x)}{sinh(x)}$	$\frac{e^x + e^{-x}}{e^x - e^{-x}}$	$i\,cot(ix)$
5. Hyperbolic Secant	$sech(x)$	$\frac{1}{cosh\,(x)}$	$\frac{2}{e^x + e^{-x}}$	$sec(ix)$
6. Hyperbolic Cosecant	$csch(x)$	$\frac{1}{sinh\,(x)}$	$\frac{2}{e^x - e^{-x}}$	$i\,csc(ix)$
7. Inverse Hyperbolic Sine	$sinh^{-1}(x)$	$ln\left(x + \sqrt{x^2 + 1}\right)$		
8. Inverse Hyperbolic Cosine	$cosh^{-1}(x)$	$ln\left(x + \sqrt{x^2 - 1}\right), \quad x \geq 1$		
9. Inverse Hyperbolic Tangent	$tanh^{-1}(x)$	$\frac{1}{2}ln\left(\frac{1+x}{1-x}\right), \quad \lvert x\rvert < 1$		
10. Inverse Hyperbolic Cotangent	$coth^{-1}(x)$	$\frac{1}{2}ln\left(\frac{x+1}{x-1}\right), \quad \lvert x\rvert > 1$		
11. Inverse Hyperbolic Secant	$sech^{-1}(x)$	$ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right), \quad 0 < x \leq 1$ $ln\left(\frac{1 + \sqrt{1 - x^2}}{x}\right), \quad 0 < x \leq 1$		
12. Inverse Hyperbolic Cosecant	$csch^{-1}(x)$	$ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1}\right), \quad x \neq 0$		
Catenary	$y = a\,cosh\left(\frac{x}{a}\right)$			

Hyperbolic Identities	
Odd and Even	$sinh(-x) = -sinh\,(x)$
	$cosh(-x) = cosh\,(x)$
	$tanh(-x) = -tanh\,(x)$
Pythagorean Identities	$\boldsymbol{cosh^2(x) - sinh^2(x) = 1}$ *(Euler's formula)*
	$sech^2(x) = 1 - tanh^2(x)$
	$csch^2(x) = coth^2(x) - 1$
Sums/Difference of Angles	$sinh(x \pm y) = sinh(x)\,cosh(y) \pm cosh(x)\,sinh(y)$
	$cosh(x \pm y) = cosh(x)\,cosh(y) \pm sinh(x)\,sinh(y)$
	$tanh(x \pm y) = \dfrac{tanh(x) \pm tanh(y)}{1 \pm tanh(x)\,tanh(y)}$
Double Angle	$sinh(2x) = 2\,sinh(x)\,cosh(x)$
	$cosh(2x) = sinh^2(x) + cosh^2(x)$ $= 2\,cosh^2(x) - 1$ $= 1 + 2\,sinh^2(x)$
	$tanh(2x) = \dfrac{2\,tanh(x)}{1 + tanh^2(x)}$
Half Angle	$sinh\left(\frac{x}{2}\right) = \dfrac{sinh(x)}{\sqrt{2(cosh(x) + 1)}}$
	$cosh\left(\frac{x}{2}\right) = \sqrt{\dfrac{cosh(x) + 1}{2}}$
	$tanh\left(\frac{x}{2}\right) = \dfrac{sinh(x)}{cosh(x) + 1}$ $= \dfrac{cosh(x) - 1}{sinh(x)}$ $= coth(x) - csch(x)\;\; if\;\; x \neq 0$
Squared	$sinh^2(x) = \frac{1}{2}(cosh(2x) - 1)$
	$cosh^2(x) = \frac{1}{2}(cosh(2x) + 1)$
Exponents (Powers)	$(cosh(x) + sinh(x))^n = cosh(nx) + sinh(nx)$
Natural Exponential	$e^x = cosh(x) + sinh(x)$
	$e^{-x} = cosh(x) - sinh(x)$
Natural Logarithmic	$ln(x) = \pm\,cosh^{-1}\left(\dfrac{x^2 + 1}{2x}\right)$ $= sinh^{-1}\left(\dfrac{x^2 - 1}{2x}\right)$ $= tanh^{-1}\left(\dfrac{x^2 - 1}{x^2 + 1}\right)$



Hyperbolic Derivatives	
1. Hyperbolic Sine	$\frac{d}{dx}[sinh(x)] = cosh(x)$
2. Hyperbolic Cosine	$\frac{d}{dx}[cosh(x)] = sinh(x)$
3. Hyperbolic Tangent	$\frac{d}{dx}[tanh(x)] = sech^2(x)$
4. Hyperbolic Cotangent	$\frac{d}{dx}[coth(x)] = -\,csch^2(x), \quad x \neq 0$
5. Hyperbolic Secant	$\frac{d}{dx}[sech(x)] = -\,sech(x)\,tanh(x)$
6. Hyperbolic Cosecant	$\frac{d}{dx}[csch(x)] = -\,csch(x)\,coth(x), \quad x \neq 0$
7. Hyperbolic Arcsine	$\frac{d}{dx}[sinh^{-1}(x)] = \frac{1}{\sqrt{x^2+1}}$
8. Hyperbolic Arccosine	$\frac{d}{dx}[cosh^{-1}(x)] = \frac{1}{\sqrt{x^2-1}}, \quad x > 1$
9. Hyperbolic Arctangent	$\frac{d}{dx}[tanh^{-1}(x)] = \frac{1}{1-x^2}, \quad \lvert x \rvert < 1$
10. Hyperbolic Arccotangent	$\frac{d}{dx}[coth^{-1}(x)] = \frac{1}{1-x^2}, \quad \lvert x \rvert > 1$
11. Hyperbolic Arcsecant	$\frac{d}{dx}[sech^{-1}(x)] = \frac{-1}{x\sqrt{1-x^2}}, \quad 0 < x < 1$
12. Hyperbolic Arccosecant	$\frac{d}{dx}[csch^{-1}(x)] = \frac{-1}{\lvert x \rvert\sqrt{1+x^2}}, \quad x \neq 0$

Hyperbolic 2nd Derivatives	
1. Hyperbolic Sine	$\frac{d^2}{dx^2}[sinh(x)] = sinh(x)$
2. Hyperbolic Cosine	$\frac{d^2}{dx^2}[cosh(x)] = cosh(x)$

Hyperbolic Integrals	(*See: Wikipedia, list of integrals of hyperbolic functions.*)
1. Hyperbolic Sine	$\int sinh(x)\,dx = cosh(x) + C$
2. Hyperbolic Cosine	$\int cosh(x)\;dx = sinh(x) + C$
3. Hyperbolic Tangent	$\int tanh(x)\,dx = ln\lvert cosh\,(x)\rvert + C$
4. Hyperbolic Cotangent	$\int coth(x)\,dx = ln\lvert sinh\,(x)\rvert + C$
5. Hyperbolic Secant	$\int sech(x)\,dx = ln\lvert sech(x) + tanh(x)\rvert$ $= tanh^{-1}(sinh(x)) + C$ $= 2\,tan^{-1}\left\lvert tanh\left(\frac{x}{2}\right)\right\rvert + C$
6. Hyperbolic Cosecant	$\int csch(x)\,dx = ln\lvert coth(x) - csch(x)\rvert + C$ $= coth^{-1}(cosh(x)) + C$ $= ln\left\lvert tanh\left(\frac{x}{2}\right)\right\rvert$
7. Hyperbolic Secant²	$\int sech^2(x)\,dx = tanh(x) + C$
8. Hyperbolic Cosecant²	$\int cosh^2(x)\,dx = -\,coth(x) + C$
9. Hyperbolic Arcsine	$\int sinh^{-1}\left(\frac{x}{c}\right)\;dx = x\;sinh^{-1}\left(\frac{x}{c}\right) - \sqrt{x^2 + c^2} + C$
10. Hyperbolic Arccosine	$\int cosh^{-1}\left(\frac{x}{c}\right)\;dx = x\;cosh^{-1}\left(\frac{x}{c}\right) - \sqrt{x^2 - c^2} + C$
11. Hyperbolic Arctangent	$\int tanh^{-1}\left(\frac{x}{c}\right)\;dx = x\;tanh^{-1}\left(\frac{x}{c}\right) + \frac{c}{2}ln\lvert c^2 - x^2\rvert + C,$ $\lvert x\rvert < \lvert c\rvert$
12. Hyperbolic Arccotangent	$\int coth^{-1}\left(\frac{x}{c}\right)\;dx = x\;coth^{-1}\left(\frac{x}{c}\right) + \frac{c}{2}ln\lvert x^2 - c^2\rvert + C,$ $\lvert x\rvert > \lvert c\rvert$
13. Hyperbolic Arcsecant	$\int sech^{-1}\left(\frac{x}{c}\right)\;dx = x\;sech^{-1}\left(\frac{x}{c}\right) - c\,tan^{-1}\frac{x\sqrt{\frac{c-x}{c+x}}}{x-c} + C,$ $0 < x < c$
14. Hyperbolic Arccosecant	$\int csch^{-1}\left(\frac{x}{c}\right)\;dx = x\;csch^{-1}\left(\frac{x}{c}\right) + c\,ln\frac{x + \sqrt{x^2 + c^2}}{c} + C,\quad 0$ $< x < c$
9. Hyperbolic Arcsine	$\int \frac{1}{\sqrt{a^2 + x^2}}\;dx = sinh^{-1}\left(\frac{x}{c}\right) + C$
10. Hyperbolic Arccosine	$\int \frac{1}{\sqrt{x^2 - a^2}}\;dx = cosh^{-1}\left\lvert\frac{x}{c}\right\rvert + C$

11. Hyperbolic Arctangent	$\int \frac{1}{a^2 - x^2}\, dx = \frac{1}{a}\, tanh^{-1}\left(\frac{x}{c}\right) + C, \qquad x^2 < a^2$
12. Hyperbolic Arccotangent	$\int \frac{1}{a^2 - x^2}\, dx = \frac{1}{a}\, coth^{-1}\left(\frac{x}{c}\right) + C, \qquad x^2 > a^2$
13. Hyperbolic Arcsecant	$\int \frac{1}{x\,\sqrt{a^2 - x^2}}\, dx = -\frac{1}{a}\, sech^{-1}\left\|\frac{x}{c}\right\| + C$
14. Hyperbolic Arccosecant	$\int \frac{1}{x\,\sqrt{a^2 + x^2}}\, dx = -\frac{1}{a}\, csch^{-1}\left\|\frac{x}{c}\right\| + C$

Sources

- Wikipedia (2025). Hyperbolic Functions. https://en.wikipedia.org/wiki/Hyperbolic_functions.

See Also

- Harold’s Trigonometry & Hyperbolic Parent Functions Cheat Sheet