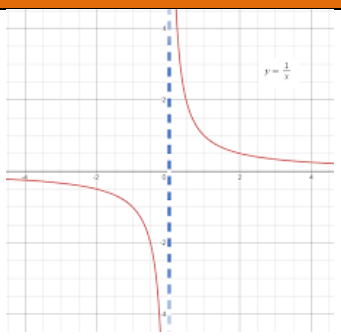
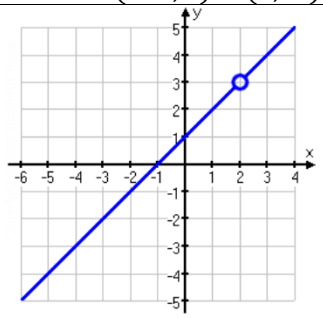
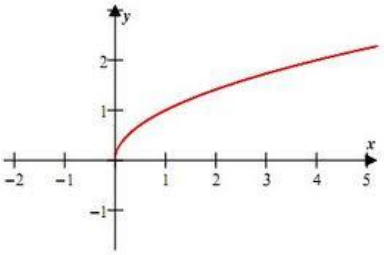
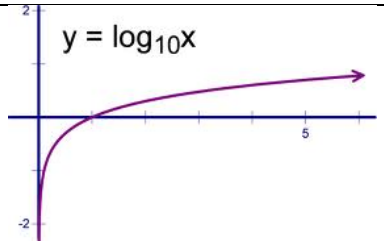
Harold's Illegals
Cheat Sheet

AKA Undefined Operations and Indeterminate Forms

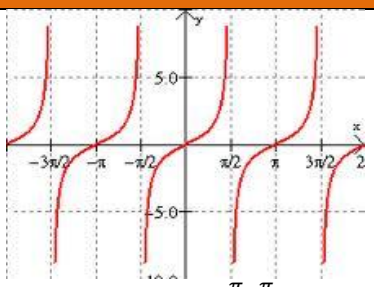
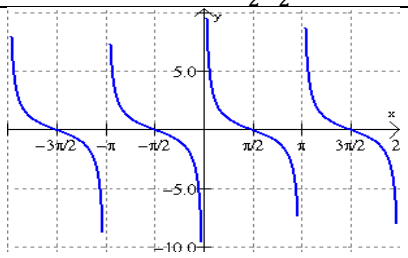
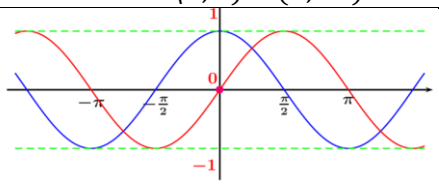
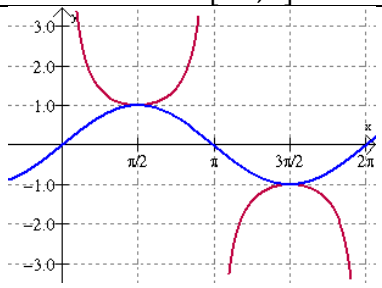
5 May 2026

Algebra

Function	What is Undefined?	When is it Defined?	Graph
$\frac{1}{x}$	Division by zero (0) (vertical asymptote)	$x \neq 0$	 $Domain$: $(-\infty, 0) \cup (0, \infty)$
$f(x)\frac{(x-c)}{(x-c)}$	Division by zero (0) – special case (hole at $x = c$)	$x \neq c$	 $Domain$: $(-\infty, c) \cup (c, \infty)$
$\sqrt{x}$	Negative square roots	$x \geq 0$	 *Domain:* $[0, \infty)$
$\sqrt[2n]{x} = x^{\frac{1}{2n}}$	Negative even-powered roots	$x \geq 0$	*Domain:* $[0, \infty)$
$\log x$ $\ln x$	Negative logarithms	$x > 0$	 *Domain:* $(0, \infty)$

Copyright © 2014-2026 by Harold Toomey, WyzAnt Tutor

Trigonometry

Function	What is Undefined?	When is it Defined?	Graph
$\tan x$ $\sec x$	Trig functions with division by zero when $\cos x = 0$	$x \neq \frac{\pi}{2} \pm n\pi$	 *Domain:* $(-\frac{\pi}{2}, \frac{\pi}{2})$
$\cot x$ $\csc x$	Trig functions with division by zero when $\sin x = 0$	$x \neq \pm n\pi$	 *Domain:* $(0, \pi) \cup (\pi, 2\pi)$
$\sin^{-1} x$ $\cos^{-1} x$	Inverse trig functions where $\lvert x \rvert > 1$	$\lvert x \rvert \leq 1$	 *Domain*: $[-1, 1]$
$\sec^{-1} x$ $\csc^{-1} x$	Inverse trig functions where $\lvert x \rvert < 1$	$\lvert x \rvert \geq 1$	 *Domain*: $(-\infty, -1] \cup [1, \infty)$

Copyright © 2014-2026 by Harold A. Toomey, WyzAnt Tutor

Zero/Infinity

Number	What is Undefined?	When is it Defined?
Zero (0) Infinity (∞)	$\frac{0}{0}, \frac{\infty}{\infty}, 0 \bullet \infty, \infty - \infty, 0^0, \infty^0, 1^\infty, x^\infty$ Incorrect Math: 1. $\frac{0}{0} = 1$ 2. $\infty^0 = 1$ 3. $\infty - \infty = \infty$ 4. $1^\infty = 1$ 5. $0^0 = 1$ 6. $\frac{\infty}{\infty} = \infty$ 7. $0 \cdot \infty = 0$	Correct Math: $0^\infty \to 0$ $\infty^\infty \to \infty$ $\frac{0}{\infty} \to 0$ $\frac{\infty}{0} \to \infty$
Infinity (∞)	Treating infinity like a number. $f(\infty)$	$\lim_{x \to \infty} f(x)$
Proof $2 = 1$	Given: $a = b$ Proof: $a = b$ $a^2 = a \cdot b$ $a^2 - b^2 = a \cdot b - b^2$ $(a + b)(a - b) = b(a - b)$ $a + b = b$ $b + b = b$ $2b = b$ $2 = 1$	Can you find the illegal operation?

NOTE: Illegal = Undefined expression per Wikipedia

Copyright © 2014-2026 by Harold A. Toomey, WyzAnt Tutor