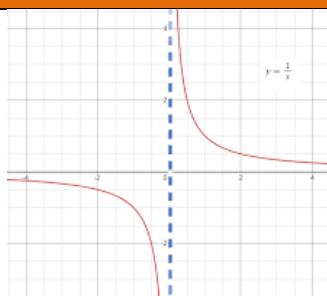
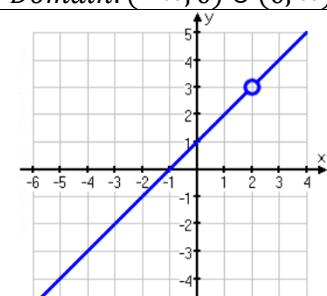
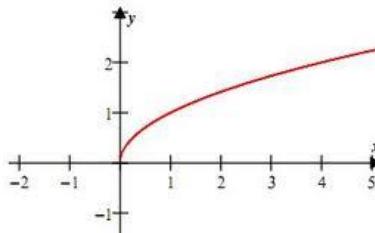
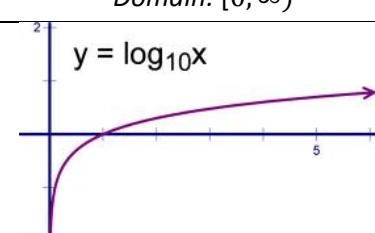


# Harold's Illegals

## Cheat Sheet

18 August 2025

### Algebra

Function	What is Undefined?	When is it Defined?	Graph
$\frac{1}{x}$	Division by zero (0) (vertical asymptote)	$x \neq 0$	 <p><math>y = \frac{1}{x}</math></p> <p>Domain: <math>(-\infty, 0) \cup (0, \infty)</math></p>
$f(x) \frac{(x - c)}{(x - c)}$	Division by zero (0) – special case (hole at $x = c$ )	$x \neq c$	 <p>Domain: <math>(-\infty, c) \cup (c, \infty)</math></p>
$\sqrt{x}$	Negative square roots	$x \geq 0$	 <p>Domain: <math>[0, \infty)</math></p>
$\sqrt[2n]{x} = x^{\frac{1}{2n}}$	Negative even-powered roots	$x \geq 0$	<p>Domain: <math>[0, \infty)</math></p>
$\log x$ $\ln x$	Negative logarithms	$x > 0$	 <p><math>y = \log_{10}x</math></p> <p>Domain: <math>(0, \infty)</math></p>

## Trigonometry

Function	What is Undefined?	When is it Defined?	Graph
$\tan x$ $\sec x$	Trig functions with division by zero when $\cos x = 0$	$x \neq \frac{\pi}{2} + n\pi$	<p>Domain: <math>(-\frac{\pi}{2}, \frac{\pi}{2})</math></p>
$\cot x$ $\csc x$	Trig functions with division by zero when $\sin x = 0$	$x \neq \pm n\pi$	<p>Domain: <math>(0, \pi) \cup (\pi, 2\pi)</math></p>
$\sin^{-1} x$ $\cos^{-1} x$	Inverse trig functions where $ x  > 1$	$ x  \leq 1$	<p>Domain: <math>[-1, 1]</math></p>
$\sec^{-1} x$ $\csc^{-1} x$	Inverse trig functions where $ x  < 1$	$ x  \geq 1$	<p>Domain: <math>(-\infty, -1] \cup [1, \infty)</math></p>

## Zero/Infinity

Number	What is Undefined?	When is it Defined?
Zero (0) Infinity ( $\infty$ )	$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, \infty - \infty, 0^0, \infty^0, 1^\infty, x^\infty$	$0^\infty \rightarrow 0, \quad \infty^\infty \rightarrow \infty$ $\frac{0}{\infty} \rightarrow 0, \quad \frac{\infty}{0} \rightarrow \infty$
Infinity ( $\infty$ )	Treating infinity like a number. $f(\infty)$	$\lim_{x \rightarrow \infty} f(x)$
Proof $2 = 1$	<p>Given:  <math>a = b</math></p> <p>Proof:  <math>a = b</math>  <math>a^2 = a \cdot b</math>  <math>a^2 - b^2 = a \cdot b - b^2</math>  <math>(a + b)(a - b) = b(a - b)</math>  <math>a + b = b</math>  <math>b + b = b</math>  <math>2b = b</math>  <math>2 = 1</math></p>	Can you find the illegal operation?

NOTE: Illegal = [Undefined expression](#) per Wikipedia