**Harold’s Matrix Cheat Sheet**

16 March 2024

**Matrix Definitions**

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| **Property** | **Example** |
| **Dimension** | A number and arrows with text  Description automatically generated with medium confidence$$Dim=rows ×columns=2×3$$ |
| **Vector** | $n×1$ Matrix |
| **Zero Matrix** | $$O=\left[\begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix}\right]$$ |
| **Identity Matrix (**$I\_{n}$**)** | A number and numbers in a row  Description automatically generated with medium confidence |
| **Matrix Elements** |  |
| **Rank** |  |
| **Row Matrix** | A Matrix in which there is only one row and no column. |
| **Column Matrix** | A Matrix in which there is only one column and no row. |
| **Horizontal Matrix** | A Matrix in which the number of rows is less than the number of columns. |
| **Vertical Matrix** | A Matrix in which the number of columns is less than the number of rows. |
| **Rectangular Matrix** | A Matrix in which the number of rows and columns are unequal. |
| **Square Matrix** | A matrix in which the number of rows and columns are the same. |
| **Diagonal Matrix** | A square matrix in which the non-diagonal elements are zero. |
| **Zero or Null Matrix** | A matrix whose all elements are zero. |
| **Unit or Identity Matrix** | A diagonal matrix whose all diagonal elements are 1.  |
| **Symmetric Matrix** | A square matrix where the transpose of the original matrix is equal to its original matrix. i.e. $(A^{T})=A$. |
| Skew-symmetric Matrix | A skew-symmetric (or antisymmetric or antimetric[1]) matrix is a square matrix whose transpose equals its negative i.e. $\left(A^{T}\right)=-A$.  |
| Orthogonal Matrix | $$AA^{T}=A^{T}A=I\_{n}$$ |
| Idempotent Matrix | $$A^{2}=A$$ |
| Involutory Matrix | $$A^{2}=I\_{n}$$ |
| Upper Triangular Matrix | A square matrix in which all the elements below the diagonal are zero. |
| Lower Triangular Matrix | A square matrix in which all the elements above the diagonal are zero. |
| Singular Matrix | A square matrix whose determinant is zero. i.e. $|A|=0$ |
| Nonsingular Matrix | A square matrix whose determinant is non-zero. i.e. $|A|\ne 0$ |

**Matrix Properties**

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| **Property** | **Example** |
| **Matrix Addition** |
| Commutative | $$A+B=B+A$$ |
| Associative | $$\left(A+B\right)+C=A+\left(B+C\right)$$ |
| Additive Identity | For any matrix *A*, there is a unique matrix *O* such that$A+0=A$. |
| Additive Inverse | For each $A$, there is a unique matrix $-A$ such that$A+\left(-A\right)=0$. |
| Closure | $A+B$ is a matrix of the same dimensions as $A$ and $B$. |
| **Scalar Multiplication** |
| Associative | $$\left(cd\right)A=c(dA)$$$$c\left(AB\right)=\left(cA\right)B=A(cB)$$ |
| Distributive | $$c\left(A+B\right)=cA+cB$$$$\left(c+d\right)A=cA+dA$$ |
| Multiplicative Identity | $$1A=A$$ |
| Multiplicative Properties of Zero | $$0∙A=O$$$$c∙O=O$$ |
| Closure | $cA$ is a matrix of the same dimensions as $A$. |
| **Matrix Multiplication** |
| Not Commutative | $$AB\ne BA$$ |
| Associative | $$\left(AB\right)C=A(BC)$$ |
| Distributive | $$A(B+C)=AB+AC$$$$\left(B+C\right)A=BA+CA$$ |
| Multiplicative Identity | $$I\_{n}A=A$$$$AI\_{n}=A$$ |
| Multiplicative Property of Zero | $$OA=O$$$$AO=O$$ |
| Dimension | The product of an $m × n$ matrix and an $n × k$ matrix is an $m×k$ matrix. |
| **Transpose** |
| Inverse | $$(A^{T})^{T}=A$$ |
| Addition | $$(A+B)^{T}=A^{T}+B^{T}$$ |
| Constant Multiple | $$(cA)^{T}=cA^{T}$$ |
| Multiplication | $(AB)^{T}=B^{T}A^{T}$ (Note reverse order) |
| Identity | $$I\_{n}^{T}=I\_{n}$$ |
| **Inverse (Square Matrix)** |
| Inverse | $$(A^{-1})^{-1}=A$$$$AA^{-1}=A^{-1}A=I\_{n}$$ |
| Distributuve | $$(cA)^{-1}=c^{-1}A^{-1}, r\ne 0$$ |
| Multiplication | $(AB)^{-1}=B^{-1}A^{-1}$ (Note reverse order, $A$ and $B$ must be invertable) |
| Identity | $$I\_{n}^{-1}=I\_{n}$$ |
| Commutative | $$(A^{T})^{-1}=(A^{-1})^{T}$$ |
| **Adjoint (Square Matrix)** |
|  | $$A\left(Adj (A)\right)=\left(Adj (A)\right)A=\left|A\right|I\_{n}$$ |
|  | $$Adj\left(AB\right)=\left(Adj (B)\right)∙(Adj (A))$$ |
|  | $$\left|Adj (A)\right|=\left|A\right|^{n-1}$$ |
|  | $$Adj\left(kA\right)=k^{n-1}Adj(A)$$ |
|  | $$\left|Adj\left(Adj\left(A\right)\right)\right|=\left|A\right|(n-1)^{2}$$ |
|  | $$Adj(Adj(A))=A^{(n-2)}×A$$ |
|  | $$If A=\left[l,m,n\right]then Adj\left(A\right)=[MN, LN, LM]$$ |
|  | $$Adj\left(I\_{n}\right)=I\_{n}$$ |

**Matrix Operations**

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| --- | --- |
| **Property** | **Example** |
| **Augmented Matrix** |  |
| **Transpose** |  |
| **Determinant** |  |
| **Dot Product** |  |
| **Cross Product** |  |
| **Adjoint** |  |
| **Norm** |  |
| **Eigen Values and Eigen Vectors** |  |
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Sources:

[Matrices: Definition, Properties, Types, Formulas, and Examples (geeksforgeeks.org)](https://www.geeksforgeeks.org/matrices/#operation-on-matrices)