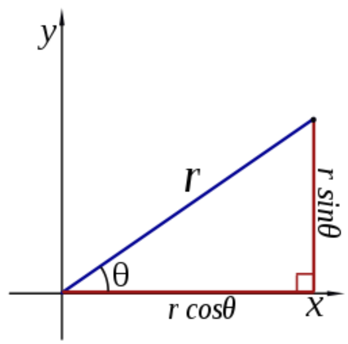
Harold's Precalculus

Cheat Sheet

24 February 2025

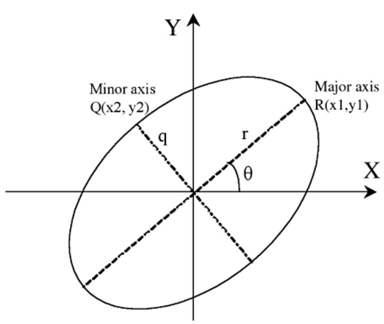
	Rectangular	Polar	Parametric
Point	$f(x) = y$ (x, y) (a, b)	(r, θ) or $r \angle \theta$ *Polar → Rect.* $x = r\cos\theta$ $y = r\sin\theta$ $\tan\theta = \frac{y}{x}$ *Rect. → Polar* $r^2 = x^2 + y^2$ $r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$	*Point (a,b) in Rectangular:* $x(t) = a$ $y(t) = b$ $< a, b >$ $t = 3^{rd}$ *variable, usually time, with 1 degree of freedom (df)*
Line	*Slope-Intercept Form:* $y = mx + b$ *Point-Slope Form:* $y - y_0 = m(x - x_0)$ *Intercept Form:* $\frac{x}{a} + \frac{y}{b} = 1$ *Normal Form:* $Ax + By + C = 0$		$< x, y > = < x_0, y_0 > + t < a, b >$ $< x, y > = < x_0 + at, y_0 + bt >$ *where* $< a, b > = < x_2 - x_1, y_2 - y_1 >$ $x(t) = x_0 + ta$ $y(t) = y_0 + tb$ $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b}{a}$
Plane	$n_x(x - x_0)$ $+ n_y(y - y_0)$ $+ n_z(z - z_0) = 0$	*Vector Form:* $\boldsymbol{n} \cdot (\boldsymbol{r} - \boldsymbol{r}_0) = 0$	$\boldsymbol{r} = \boldsymbol{r}_0 + s\boldsymbol{v} + t\boldsymbol{w}$ $x = x_0 + su_1 + tv_1$ $y = y_0 + su_2 + tv_2$ $z = z_0 + su_3 + tv_3$

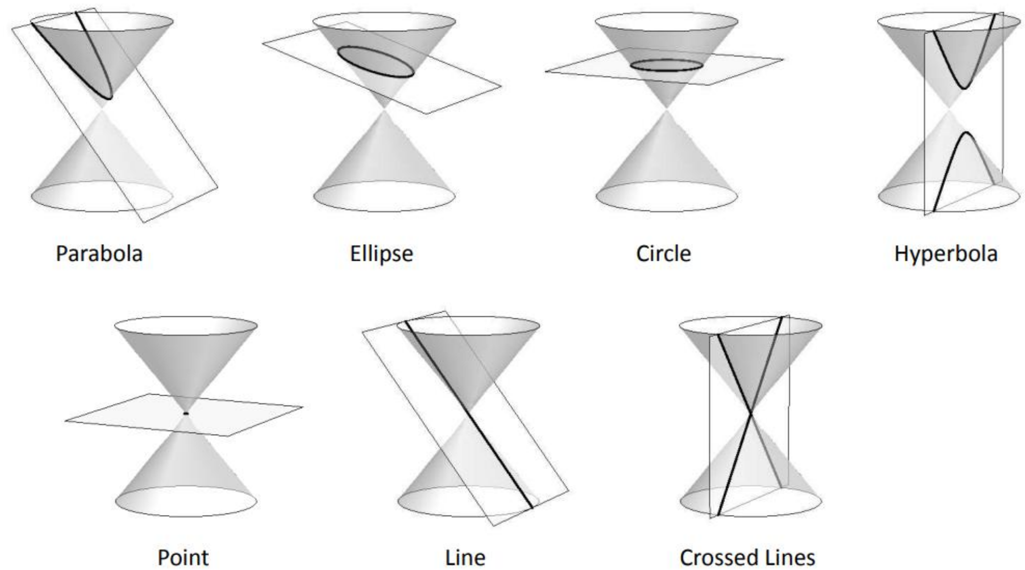
<table>
<tr><th></th><th>Rectangular</th><th>Polar</th><th>Parametric</th></tr>
<tr><td>Conics</td><td>
General Equation for All Conics:

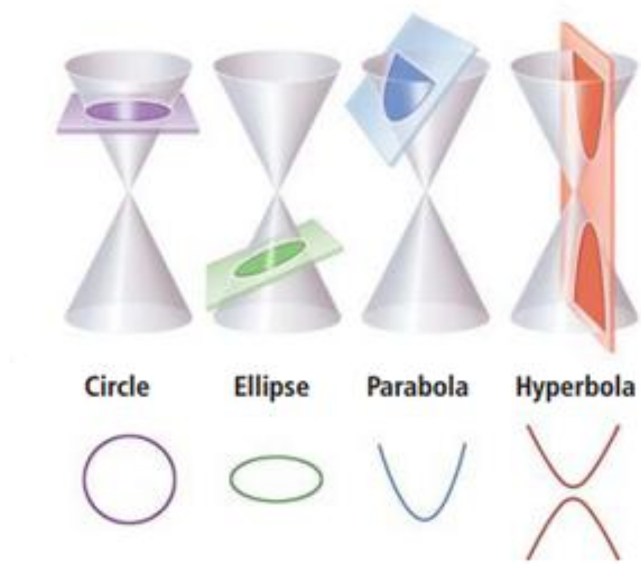
$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

where
$Line$: $A = B = C = 0$
$Circle$: $A = C \text{ and } B = 0$
$Ellipse$: $AC > 0$
$or\ B^2 - 4AC < 0$
$Parabola$: $AC = 0$
$or\ B^2 - 4AC = 0$
$Hyperbola$: $AC < 0$
$or\ B^2 - 4AC > 0$
Note: If $A + C = 0$, square hyperbola

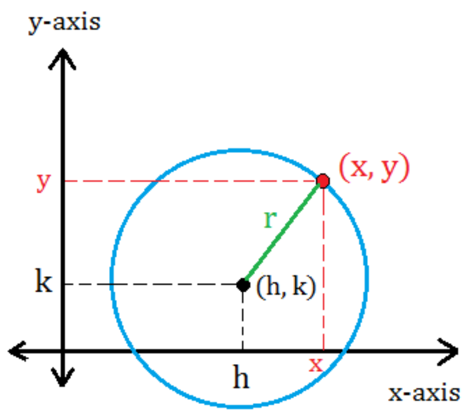
Rotation:
If B ≠ 0, then rotate the coordinate system:
$\cot 2\theta = \frac{A - C}{B}$
$x = x' \cos\theta - y' \sin\theta$
$y = y' \cos\theta + x' \sin\theta$

New = (x’, y’), Old = (x, y)
rotates through angle θ from x-axis


</td><td colspan="2">




</td></tr>
</table>

	Rectangular	Polar	Parametric
Circle	$x^2 + y^2 = r^2$ $(x-h)^2 + (y-k)^2 = r^2$ *Center:* (h,k) *Vertices: NA* *Focus:* (h,k) 	*Centered at Origin:* *r = a (constant)* $\theta \; = \; \theta \; [0, 2\pi] \; or \; [0, 360°]$	$x(t) = r \, cos(t) + h$ $y(t) = r \, sin(t) + k$ $[t_{min}, \, t_{max}] = [0, 2\pi)$ *Center:* (h,k) *Focus:* (h,k)

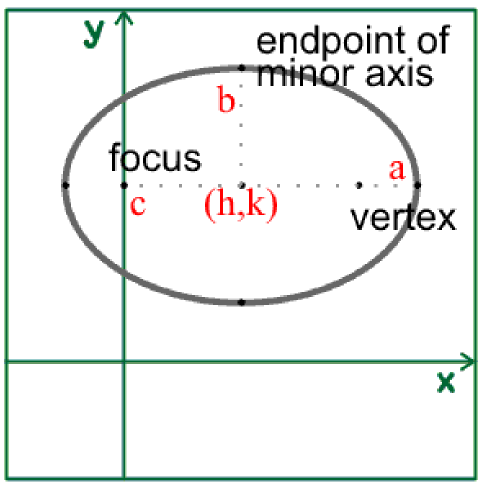
<table>
<tr><th></th><th>Rectangular</th><th>Polar</th><th>Parametric</th></tr>
<tr>
<td>Ellipse</td>
<td>

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

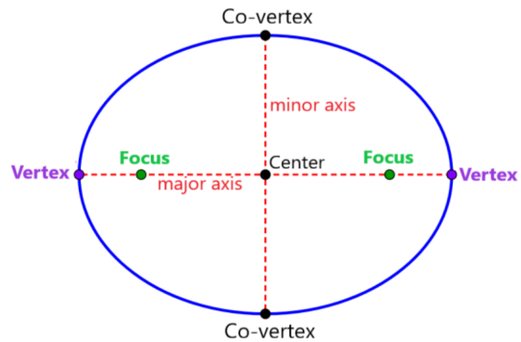
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Center: (h, k)
Vertices: $(h \pm a, k)$
Co-Vertices: $(h, k \pm b)$
Foci: $(h \pm c, k)$

Focus length, c, from center:
$c^2 = a^2 - b^2$



</td>
<td>



Interesting Note:
The <u>sum</u> of the distances from each focus to a point on the curve is constant.
$|d_1 + d_2| = k$

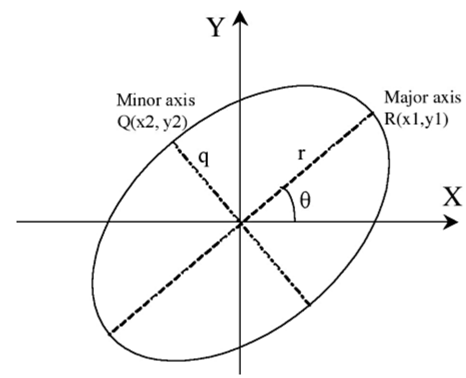
</td>
<td>

$x(t) = a\,cos(t) + h$
$y(t) = b\,sin(t) + k$
$[t_{min}, t_{max}] = [0, 2\pi]$

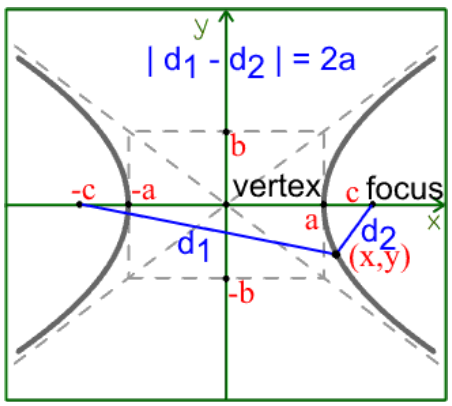
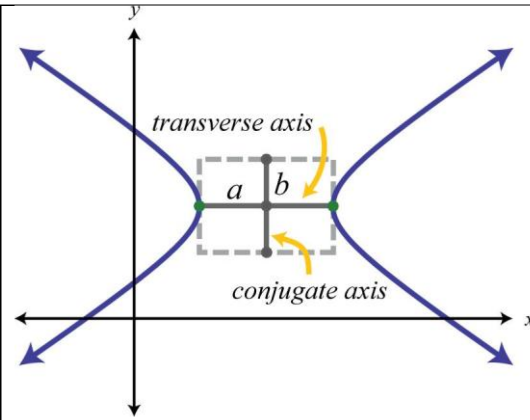
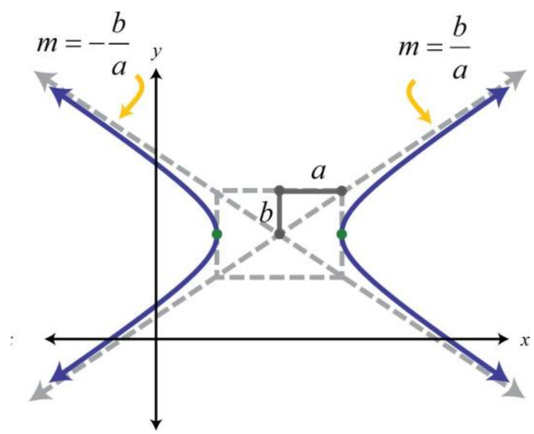
Center: (h, k)

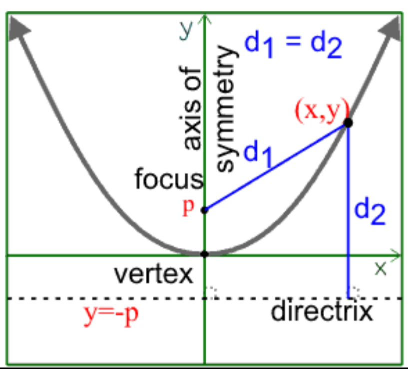
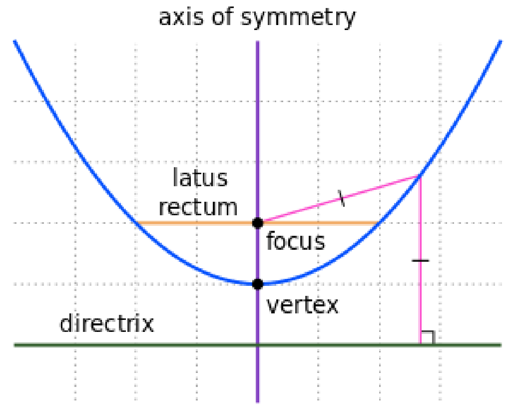
Rotated Ellipse:
$x(t) = a\cos t\cos\theta - b\sin t\sin\theta + h$
$y(t) = a\cos t\sin\theta + b\sin t\cos\theta + k$

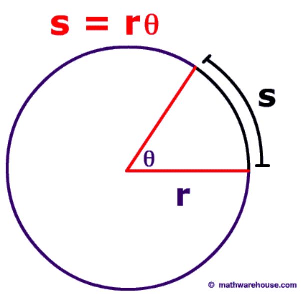
θ = the angle between the x-axis and the major axis of the ellipse



</td>
</tr>
</table>

	Rectangular	Polar	Parametric
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ *Center:* (h, k) *Vertices:* $(h \pm a, k)$ *Foci:* $(h \pm c, k)$ *Focus length, c, from center:* $c^2 = a^2 + b^2$ 	  ***Interesting Note:*** *The <u>difference</u> between the distances from each focus to a point on the curve is constant.* $\|d_1 - d_2\| = k$	*Left-Right Opening Hyperbola:* $x(t) = a\,sec(t) + h$ $y(t) = b\,tan(t) + k$ $[t_{min}, t_{max}] = [-c, c]$ *(h, k) = vertex of hyperbola* *Up-Down Opening Hyperbola:* $x(t) = a\,tan(t) + h$ $y(t) = b\,sec(t) + k$ $[t_{min}, t_{max}] = [-c, c]$ *(h, k) = vertex of hyperbola* *General Form:* $x(t) = At^2 + Bt + C$ $y(t) = Dt^2 + Et + F$ *where A and D have different signs*

	Rectangular	Polar	Parametric
Parabola	$y = ax^2 + bx + c$ $y = (x-h)^2 + k$ *Vertical Axis of Symmetry:* $x^2 = 4\,py$ $(x-h)^2 = 4p(y-k)$ *Vertex:* (h, k) *Focus:* $(h, k+p)$ *Directrix:* $y = k - p$ *Horizontal Axis of Symmetry:* $y^2 = 4\,px$ $(y-k)^2 = 4p(x-h)$ *Vertex:* (h, k) *Focus:* $(h+p, k)$ *Directrix:* $x = h - p$  	 *Vertical Axis of Symmetry:* $x(t) = 2pty = Dt^2 + Et + F$ *where A and D have the same sign* ***Interesting Note:*** *The distances from a point on the curve to the focus is the <u>same</u> as to the directrix.*	*Vertical Axis of Symmetry:* $x(t) = 2pt + h$ $y(t) = pt^2 + k$ *(opens upwards)* or $y(t) = -pt^2 - k$ *(opens downwards)* $[t_{min}, t_{max}] = [-c, c]$ *Vertex:* (h, k) *Horizontal Axis of Symmetry:* $y(t) = 2pt + k$ $x(t) = pt^2 + h$ *(opens to the right)* or $x(t) = -pt^2 - h$ *(opens to the left)* $[t_{min}, t_{max}] = [-c, c]$ *Vertex:* (h, k) *Projectile Motion:* $x(t) = x_0 + v_x t$ $y(t) = y_0 + v_y t - 16t^2$ *feet* $y(t) = y_0 + v_y t - 4.9t^2$ *meters* $v_x = v \cos\theta$ $v_y = v \sin\theta$ *General Form:* $x = At^2 + Bt + C$ $y = Dt^2 + Et + F$ *where A and D have the same sign*

	Rectangular	Polar	Parametric
Inverse Functions	$f(f^{-1}(x)) = f^{-1}(f(x)) = x$	*if* $y = sin\,\theta$ *then* $\theta = sin^{-1}y$ *if* $y = cos\,\theta$ *then* $\theta = cos^{-1}y$ *if* $y = tan\,\theta$ *then* $\theta = tan^{-1}y$ *if* $y = csc\,\theta$ *then* $\theta = csc^{-1}y$ *if* $y = sec\,\theta$ *then* $\theta = sec^{-1}y$ *if* $y = cot\,\theta$ *then* $\theta = cot^{-1}y$	*or* $\theta = \arcsin y$ *or* $\theta = \arccos y$ *or* $\theta = arctan\,y$ *or* $\theta = arccsc\,y$ *or* $\theta = arcsec\,y$ *or* $\theta = arccot\,y$
Arc Length		*Circle:* $L = s = r\theta$ *Proof:* $L = (fraction\ of\ circumference) \cdot \pi \cdot (diameter)$ $L = \left(\frac{\theta}{2\pi}\right)\pi\,(2r) = r\theta$	
Perimeter	*Square:* $P = 4s$ *Rectangle:* $P = 2l + 2w$ *Triangle:* $P = a + b + c$	*Circle:* $C = \pi d = 2\pi r$ *Ellipse:* $C \approx \pi(a+b)$	
Area	*Square:* $A = s^2$ *Rectangle:* $A = lw$ *Rhombus:* $A = ½\,ab$ *Parallelogram:* $A = Bh$ *Trapezoid:* $A = \frac{(B_1+B_2)}{2}\,h$ *Kite:* $A = \frac{d_1 d_2}{2}$	*Triangle:* $A = ½\,Bh$ *Triangle:* $A = ½\,ab\,sin(C)$ *Triangle using Heron's Formula:* $A = \sqrt{s(s-a)(s-b)(s-c)}$ $where\ s = \frac{a+b+c}{2}$ *Equilateral Triangle:* $A = \frac{\sqrt{3}}{4}s^2$	*Frustum:* $A = \frac{1}{3}\left(\frac{B_1+B_2}{2}\right)h$ *Circle:* $A = \pi r^2$ *Circular Sector:* $A = ½\,r^2\theta$ *Ellipse:* $A = \pi ab$
Lateral Surface Area	*Cylinder:* $SA = 2\pi rh$ *Cone:* $SA = \pi rl$		
Total Surface Area	*Cube:* $SA = 6s^2$ *Rectangular Box:* $SA = 2lw + 2wh + 2hl$ *Regular Tetrahedron:* $SA = 2bh$ *Cylinder:* $SA = 2\pi r\,(r + h)$	*Cone:* $SA = \pi r^2 + \pi rl = \pi r\,(r + l)$ *Sphere:* $SA = 4\pi r^2$ *Ellipsoid:* *SA = (too complex)*	

	Rectangular	Polar	Parametric
Volume	*Cube:* $V = s^3$ *Rectangular Prism:* $V = lwh$ *Cylinder:* $V = \pi r^2 h$ *Triangular Prism:* $V = Bh$ *Tetrahedron:* $V = \frac{1}{3} Bh$	*Pyramid:* $V = \frac{1}{3} Bh$ *Cone:* $V = \frac{1}{3} bh = \frac{1}{3} \pi r^2 h$ *Sphere:* $V = \frac{4}{3}\pi r^3$ *Ellipsoid:* $V = \frac{4}{3}\pi abc$	

