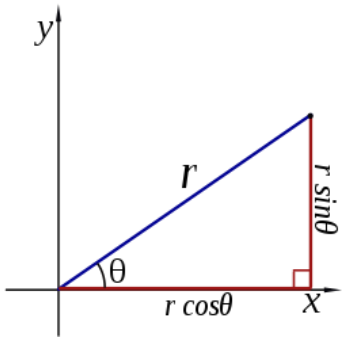
Harold's Precalculus Formulas
(Rectangular – Polar – Parametric)
Cheat Sheet

6 May 2026

	Rectangular	Polar	Parametric
Point	$f(x) = y$ (x, y) (a, b) ●	$(r\ , \theta)$ *or* $r \angle \theta$ *Polar → Rect.* $x = r\cos\theta$ $y = r\sin\theta$ $tan\,\theta = \left(\frac{y}{x}\right)$ *Rect. → Polar* $r^2 = x^2 + y^2$ $r = \sqrt{x^2 + y^2}$ $\theta = tan^{-1}\left(\frac{y}{x}\right)$	*Point (a,b) in Rectangular:* $x(t) = a$ $y(t) = b$ $< a, b >$ $t = 3^{rd}\ variable, usually\ time,$ *with 1 degree of freedom (df)*
Line	*Standard Form:* $ax + by + c = 0$ *Slope-Intercept Form:* $y = mx + b$ *Point-Slope Form:* $y - y_0 = m\,(x - x_0)$ *Intercept Form:* $\frac{x}{a} + \frac{y}{b} = 1$		$< x, y > = < x_0, y_0 > + t < a, b >$ $< x, y > = < x_0 + at, y_0 + bt >$ *where* $< a, b > = < x_2 - x_1,\ y_2 - y_1 >$ $x(t) = x_0 + ta$ $y(t) = y_0 + tb$ $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b}{a}$
Plane	$n_x(x - x_0)$ $+n_y(y - y_0)$ $+ n_z(z - z_0) = 0$	*Vector Form:* $\boldsymbol{n} \bullet (\boldsymbol{r} - \boldsymbol{r}_0) = 0$	$\boldsymbol{r} = \boldsymbol{r}_0 + s\boldsymbol{v} + t\boldsymbol{w}$ $x = x_0 + su_1 + tv_1$ $y = y_0 + su_2 + tv_2$ $z = z_0 + su_3 + tv_3$

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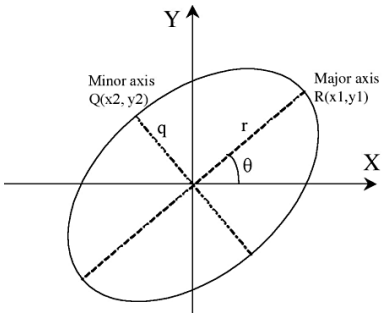
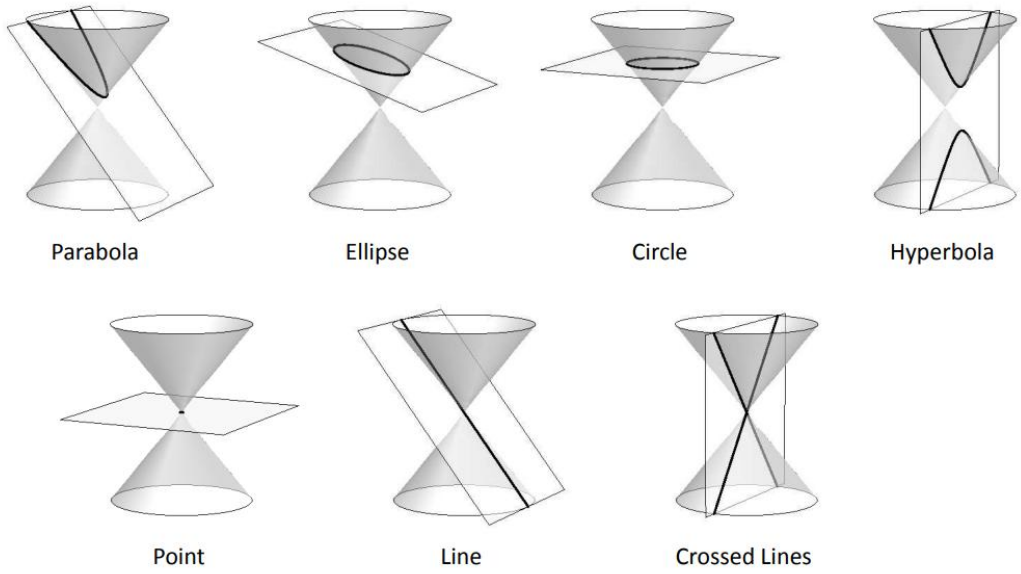
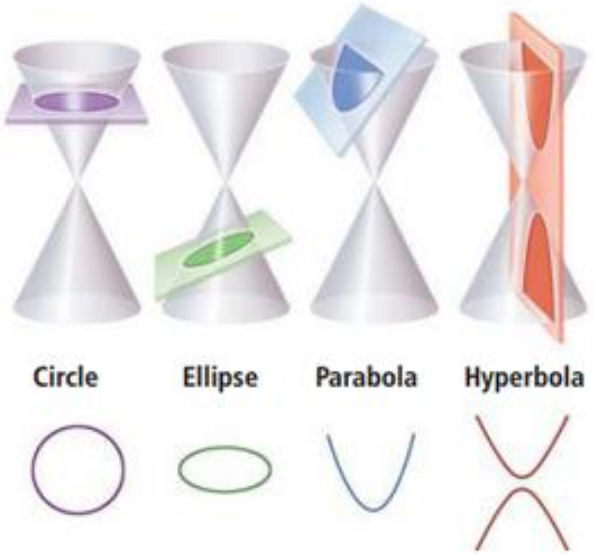
	Rectangular	Polar	Parametric
Conics	*General Equation for All Conics:* $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ *where* $Line$: $A = B = C = 0$ $Circle$: $A = C$ *and* $B = 0$ $Ellipse$: $AC > 0$ *or* $B^2 - 4AC < 0$ $Parabola$: $AC = 0$ *or* $B^2 - 4AC = 0$ $Hyperbola$: $AC < 0$ *or* $B^2 - 4AC > 0$ *Note: If* $A + C = 0$, *square hyperbola* *Rotation:* *If B ≠ 0, then rotate the coordinate system:* $\cot 2\theta = \frac{A - C}{B}$ $x = x' \cos\theta - y' \sin\theta$ $y = y' \cos\theta + x' \sin\theta$ *New = (x', y'), Old = (x, y)* *rotates through angle θ from x-axis* 	 	

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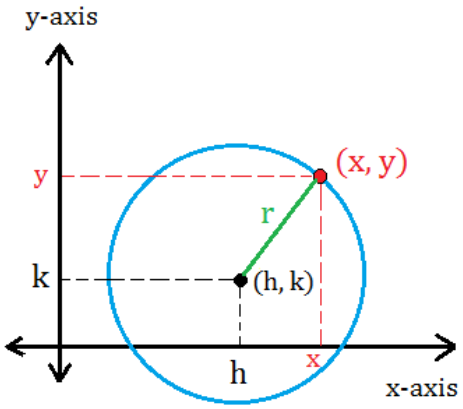
	Rectangular	Polar	Parametric
Circle	$x^2 + y^2 = r^2$ $(x-h)^2 + (y-k)^2 = r^2$ *Center:* (h, k) *Vertices: NA* *Focus:* (h, k)  	*Centered at Origin:* *r = a (constant)* $\theta = \theta\ [0, 2\pi]\ or\ [0, 360°]$	$x(t) = r\,cos(t) + h$ $y(t) = r\,sin(t) + k$ $[t_{min},\ t_{max}] = [0, 2\pi)$ *Center:* (h, k) *Focus:* (h, k)

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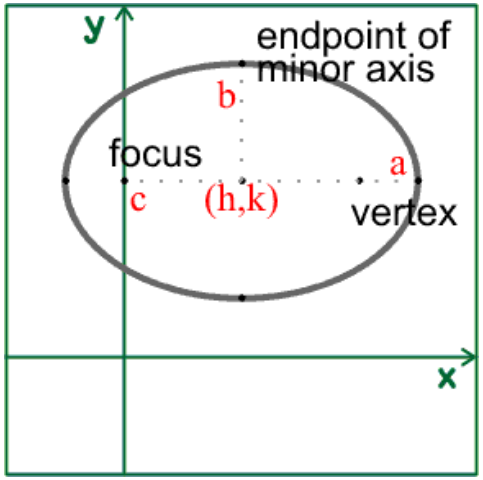
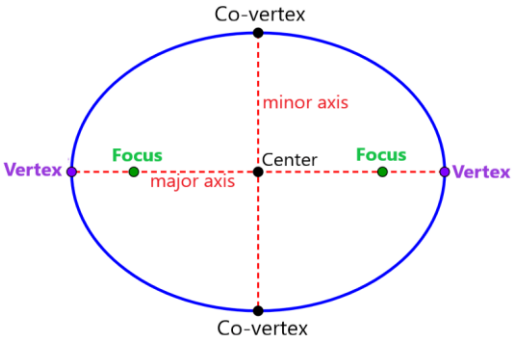
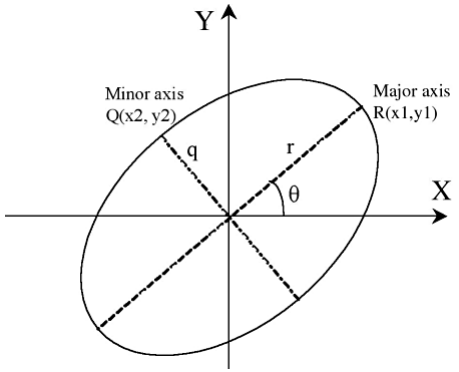
	Rectangular	Polar	Parametric
Ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ *Center:* (h,k) *Vertices:* $(h\pm a,k)$ *Co-Vertices:* $(h,k\pm b)$ *Foci:* $(h\pm c,k)$ *Focus length, c, from center:* $c^2=a^2-b^2$ 	 ***Interesting Note:*** *The <u>sum</u> of the distances from each focus to a point on the curve is constant.* $\lvert d_1+d_2\rvert=k$	$x(t)=a\,cos(t)\ +h$ $y(t)=b\,sin(t)+\ k$ $[t_{min},\ t_{max}]=[0,2\pi]$ *Center:* (h,k) *Rotated Ellipse:* $x(t)=a\cos t\cos\theta-b\sin t\sin\theta+h$ $y(t)=a\cos t\sin\theta+b\sin t\cos\theta+k$ *θ = the angle between the x-axis and the major axis of the ellipse* 

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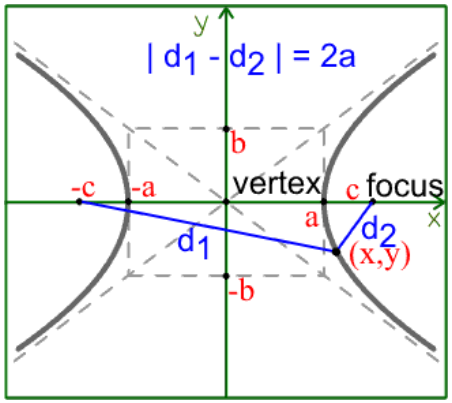
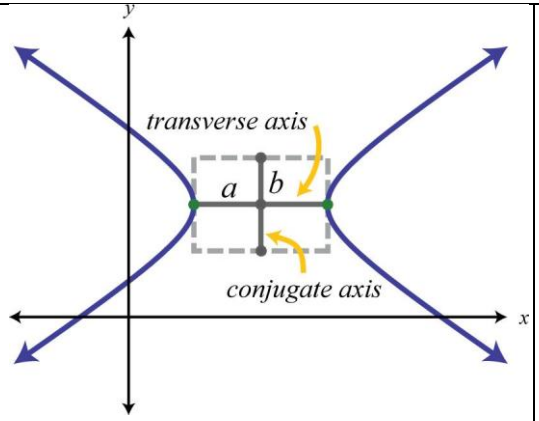
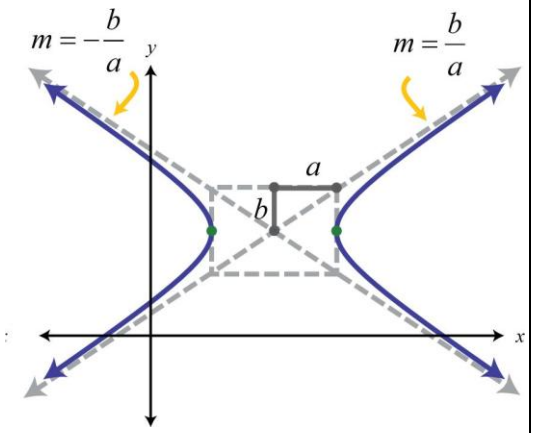
	Rectangular	Polar	Parametric
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ *Center:* (h, k) *Vertices:* $(h \pm a, k)$ *Foci:* $(h \pm c, k)$ *Focus length, c, from center:* $c^2 = a^2 + b^2$ 	  ***Interesting Note:*** *The <u>difference</u> between the distances from each focus to a point on the curve is constant.* $\|d_1 - d_2\| = k$	*Left-Right Opening Hyperbola:* $x(t) = a\,sec(t) + h$ $y(t) = b\,tan(t) + k$ $[t_{min},\ t_{max}] = [-c, c]$ *(h, k) = vertex of hyperbola* *Up-Down Opening Hyperbola:* $x(t) = a\,tan(t) + h$ $y(t) = b\,sec(t) + k$ $[t_{min},\ t_{max}] = [-c, c]$ *(h, k) = vertex of hyperbola* *General Form:* $x(t) = At^2 + Bt + C$ $y(t) = Dt^2 + Et + F$ *where A and D have different signs*

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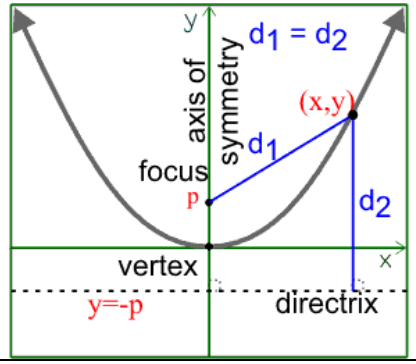
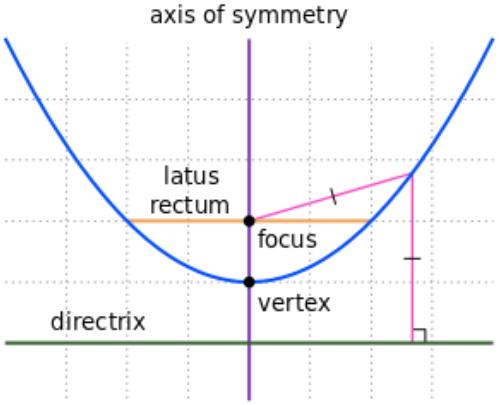
	Rectangular	Polar	Parametric
Parabola	$y = ax^2 + bx + c$ $y = (x-h)^2 + k$ *Vertical Axis of Symmetry:* $x^2 = 4\,py$ $(x-h)^2 = 4p(y-k)$ *Vertex:* (h,k) *Focus:* $(h,k+p)$ *Directrix:* $y = k-p$ *Horizontal Axis of Symmetry:* $y^2 = 4\,px$ $(y-k)^2 = 4p(x-h)$ *Vertex:* (h,k) *Focus:* $(h+p,k)$ *Directrix:* $x = h-p$ 	 *Vertical Axis of Symmetry:* $x(t) = 2pty = Dt^2 + Et + F$ *where A and D have the same sign* ***Interesting Note:*** *The distances from a point on the curve to the focus is the <u>same</u> as to the directrix.*	*Vertical Axis of Symmetry:* $x(t) = 2pt + h$ $y(t) = pt^2 + k$ *(opens upwards)* or $y(t) = -pt^2 - k$ *(opens downwards)* $[t_{min},\ t_{max}] = [-c, c]$ *Vertex:* (h,k) *Horizontal Axis of Symmetry:* $y(t) = 2pt + k$ $x(t) = pt^2 + h$ *(opens to the right)* or $x(t) = -pt^2 - h$ *(opens to the left)* $[t_{min},\ t_{max}] = [-c, c]$ *Vertex:* (h,k) *Projectile Motion:* $x(t) = x_0 + v_x t$ $y(t) = y_0 + v_y t - 16t^2$ *feet* $y(t) = y_0 + v_y t - 4.9t^2$ *meters* $v_x = v\cos\theta$ $v_y = v\sin\theta$ *General Form:* $x = At^2 + Bt + C$ $y = Dt^2 + Et + F$ *where A and D have the same sign*

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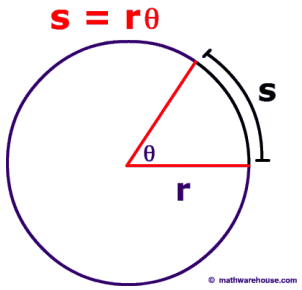
	Rectangular	Polar	Parametric
Inverse Functions	$f(f^{-1}(x)) = f^{-1}(f(x)) = x$	*if* $y = \sin\theta$ *then* $\theta = \sin^{-1} y$ *or* $\theta = \arcsin y$ *if* $y = \cos\theta$ *then* $\theta = \cos^{-1} y$ *or* $\theta = \arccos y$ *if* $y = \tan\theta$ *then* $\theta = \tan^{-1} y$ *or* $\theta = \arctan y$ *if* $y = \csc\theta$ *then* $\theta = \csc^{-1} y$ *or* $\theta = \operatorname{arccsc} y$ *if* $y = \sec\theta$ *then* $\theta = \sec^{-1} y$ *or* $\theta = \operatorname{arcsec} y$ *if* $y = \cot\theta$ *then* $\theta = \cot^{-1} y$ *or* $\theta = \operatorname{arccot} y$	
Arc Length		*Circle:* $L = s = r\theta$ *Proof:* $L = (fraction\ of\ circumference) \cdot \pi \cdot (diameter)$ $L = \left(\frac{\theta}{2\pi}\right)\pi\,(2r) = r\theta$	
Perimeter	*Square:* $P = 4s$ *Rectangle:* $P = 2l + 2w$ *Triangle:* $P = a + b + c$	*Circle:* $C = \pi d = 2\pi r$ *Ellipse:* $C \approx \pi(a + b)$	
Area	*Square:* $A = s^2$ *Rectangle:* $A = lw$ *Rhombus:* $A = \frac{1}{2}ab$ *Parallelogram:* $A = Bh$ *Trapezoid:* $A = \frac{(B_1+B_2)}{2}h$ *Kite:* $A = \frac{d_1 d_2}{2}$	*Triangle:* $A = \frac{1}{2}Bh$ *Triangle:* $A = \frac{1}{2}ab\sin(C)$ *Triangle using Heron's Formula:* $A = \sqrt{s(s-a)(s-b)(s-c)}$ $where\ s = \frac{a+b+c}{2}$ *Equilateral Triangle:* $A = \frac{\sqrt{3}}{4}s^2$	*Frustum:* $A = \frac{1}{3}\left(\frac{B_1+B_2}{2}\right)h$ *Circle:* $A = \pi r^2$ *Circular Sector:* $A = \frac{1}{2}r^2\theta$ *Ellipse:* $A = \pi ab$
Lateral Surface Area	*Cylinder:* $SA = 2\pi rh$ *Cone:* $SA = \pi rl$		
Total Surface Area	*Cube:* $SA = 6s^2$ *Rectangular Box:* $SA = 2lw + 2wh + 2hl$ *Regular Tetrahedron:* $SA = 2bh$ *Cylinder:* $SA = 2\pi r\,(r + h)$	*Cone:* $SA = \pi r^2 + \pi rl = \pi r\,(r + l)$ *Sphere:* $SA = 4\pi r^2$ *Ellipsoid:* *SA = (too complex)*	

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	Rectangular		Polar		Parametric
Volume	*Cube:*	$V = s^3$	*Pyramid:*	$V = \frac{1}{3} Bh$	
	Rectangular Prism:	$V = lwh$	*Cone:*	$V = \frac{1}{3} bh = \frac{1}{3} \pi r^2 h$	
	Cylinder:	$V = \pi r^2 h$	*Sphere:*	$V = \frac{4}{3} \pi r^3$	
	Triangular Prism:	$V = Bh$	*Ellipsoid:*	$V = \frac{4}{3} \pi abc$	
	Tetrahedron:	$V = \frac{1}{3} Bh$			

AP Precalculus Exam

Multiple Choice Weighting

Units	Exam Weighting
Unit 1: Polynomial and Rational Functions	30–40%
Unit 2: Exponential and Logarithmic Functions	27–40%
Unit 3: Trigonometric and Polar Functions	30–35%
Unit 4: Functions Involving Parameters, Vectors, and Matrices	0%
Mathematical Practices Key	
1	Procedural and Symbolic Fluency
2	Multiple Representations
3	Communication and Reasoning

Course at a Glance

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	Rectangular	Polar	Parametric

UNIT 1 Polynomial and Rational Functions

6–8 weeks | 30–40% AP Exam Weighting

- 1.1 Change in Tandem
- 1.2 Rates of Change
- 1.3 Rates of Change in Linear and Quadratic Functions
- 1.4 Polynomial Functions and Rates of Change
- 1.5 Polynomial Functions and Complex Zeros
- 1.6 Polynomial Functions and End Behavior
- 1.7 Rational Functions and End Behavior
- 1.8 Rational Functions and Zeros
- 1.9 Rational Functions and Vertical Asymptotes
- 1.10 Rational Functions and Holes
- 1.11 Equivalent Representations of Polynomial and Rational Expressions
- 1.12 Transformations of Functions
- 1.13 Function Model Selection and Assumption Articulation
- 1.14 Function Model Construction and Application

UNIT 2 Exponential and Logarithmic Functions

6–9 weeks | 27–40% AP Exam Weighting

- 2.1 Change in Arithmetic and Geometric Sequences
- 2.2 Change in Linear and Exponential Functions
- 2.3 Exponential Functions
- 2.4 Exponential Function Manipulation
- 2.5 Exponential Function Context and Data Modeling
- 2.6 Competing Function Model Validation
- 2.7 Composition of Functions
- 2.8 Inverse Functions
- 2.9 Logarithmic Expressions
- 2.10 Inverses of Exponential Functions
- 2.11 Logarithmic Functions
- 2.12 Logarithmic Function Manipulation
- 2.13 Exponential and Logarithmic Equations and Inequalities
- 2.14 Logarithmic Function Context and Data Modeling
- 2.15 Semi-log Plots

UNIT 3 Trigonometric and Polar Functions

7–10 weeks | 30–35% AP Exam Weighting

- 3.1 Periodic Phenomena
- 3.2 Sine, Cosine, and Tangent
- 3.3 Sine and Cosine Function Values
- 3.4 Sine and Cosine Function Graphs
- 3.5 Sinusoidal Functions
- 3.6 Sinusoidal Function Transformations
- 3.7 Sinusoidal Function Context and Data Modeling
- 3.8 The Tangent Function
- 3.9 Inverse Trigonometric Functions
- 3.10 Trigonometric Equations and Inequalities
- 3.11 The Secant, Cosecant, and Cotangent Functions
- 3.12 Equivalent Representations of Trigonometric Functions
- 3.13 Trigonometry and Polar Coordinates
- 3.14 Polar Function Graphs
- 3.15 Rates of Change in Polar Functions

UNIT 4 Functions Involving Parameters, Vectors, and Matrices

7 weeks | 0% AP Exam Weighting

- 4.1 Parametric Functions
- 4.2 Parametric Functions Modeling Planar Motion
- 4.3 Parametric Functions and Rates of Change
- 4.4 Parametrically Defined Circles and Lines
- 4.5 Implicitly Defined Functions
- 4.6 Conic Sections
- 4.7 Parametrization of Implicitly Defined Functions
- 4.8 Vectors
- 4.9 Vector-Valued Functions
- 4.10 Matrices
- 4.11 The Inverse and Determinant of a Matrix
- 4.12 Linear Transformations and Matrices
- 4.13 Matrices as Functions
- 4.14 Matrices Modeling Contexts

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