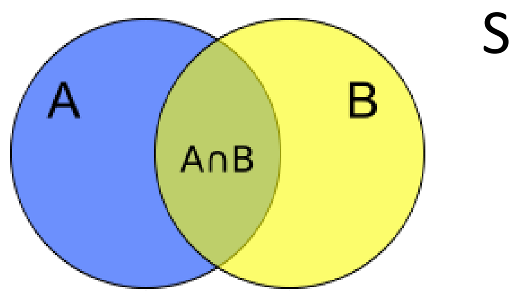
Harold’s Probability
Cheat Sheet
11 December 2024

Probability

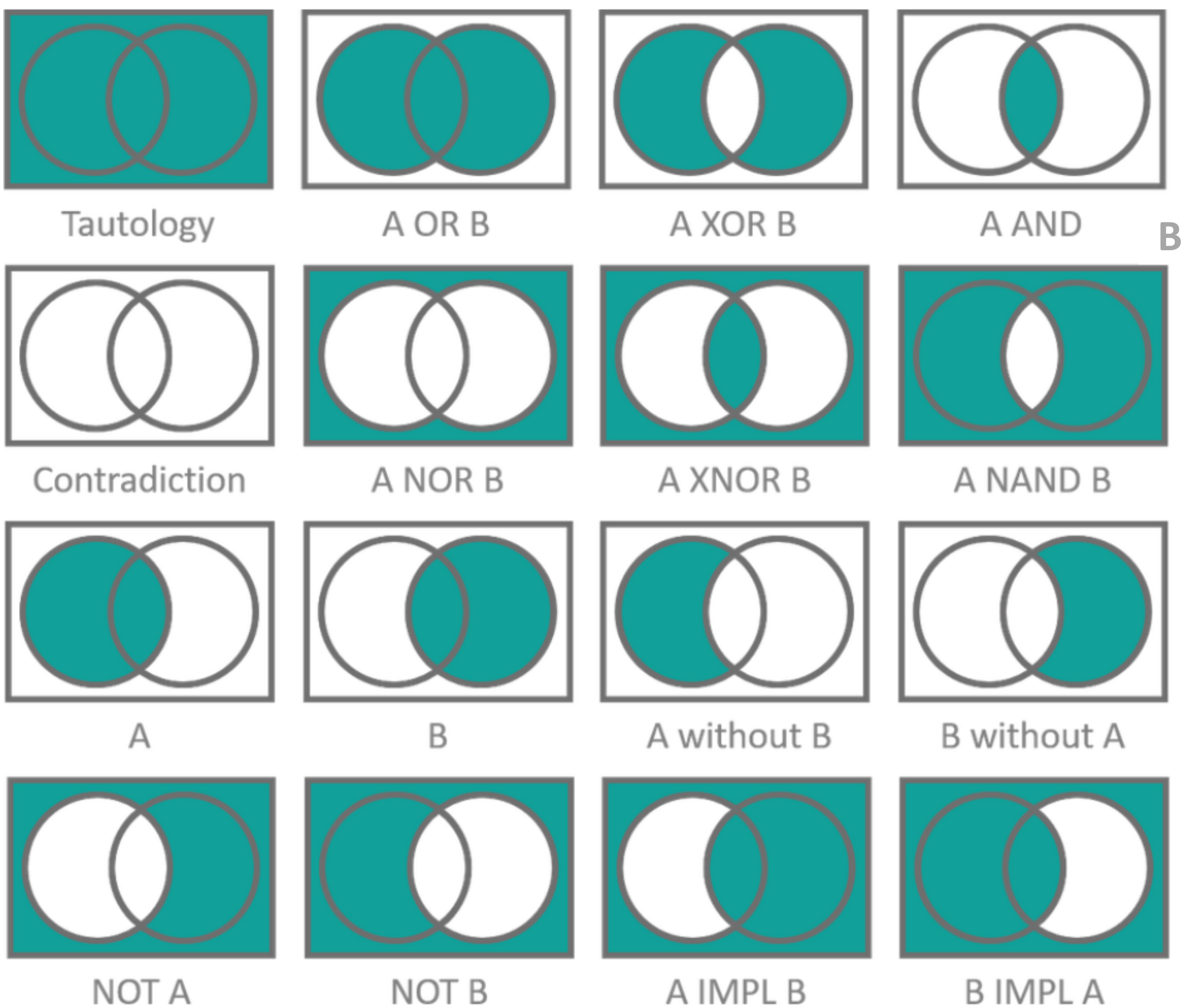
Rule	Formula	Definition
Notation	∩ = “and”, Intersection, “∧” ∪ = “or”, Union, “∨” $\overline{\ }$ = “not”, negation, “¬”	“and” implies multiplication. “or” implies addition. “not” implies negation.
Independent	If $P(A\|B) = P(A)$	The occurrence of one event does not affect the probability of the other, or vice versa.
Dependent	If $P(A \cap B) \neq \emptyset$	The occurrence of one event affects the probability of the other event.
Disjoint (“mutually exclusive”)	If $P(A \cap B) = \emptyset$ Then $\boldsymbol{P(A \cup B) = P(A) + P(B)}$	The events can never occur together.
Probability (“likelihood”)	$\boldsymbol{0 \leq P(E) \leq 1}$ $P(E) = \dfrac{\#\ Events\ (E)}{Sample\ Space\ (S)} = \dfrac{\#\ of\ Favorable\ Outcomes}{Total\ \#\ of\ Possible\ Outcomes}$	
Addition Rule (“or”)	$\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$	

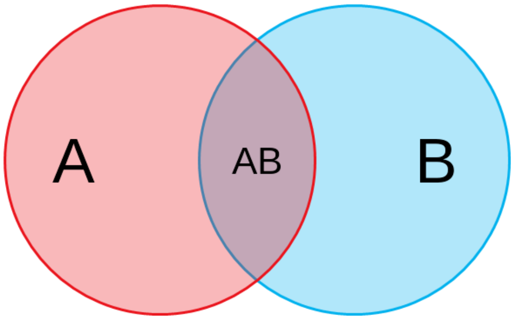
Multiplication Rule
(“and”)
	if independent or disjoint:
	$\boldsymbol{P(A \cap B) = P(A)\,P(B)}$
	$\boldsymbol{P(A \cap B \cap C) = P(A)\,P(B)\,P(C)}$
	
	if dependent:
	$\boldsymbol{P(A \cap B) = P(A)\,P(B\|A)}$
	$P(A \cap B) = P(B)\,P(A\|B)$
	$P(A \cap B) = P(A) - P\left(A \cap \overline{B}\right)$	

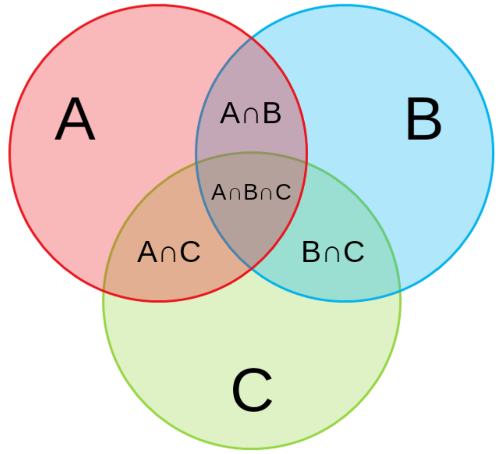
Complement Rule / Subtraction Rule (“not”)	$\boldsymbol{P(S)} = P\left(A \cup \overline{A}\right) =$ $P(A) + P\left(\overline{A}\right) = \boldsymbol{1}$ $P(A) = 1 - P\left(\overline{A}\right)$ $\boldsymbol{P\left(\overline{A}\right) = 1 - P(A)}$ $P(A\|B) + P\left(\overline{A}\|B\right) = 1$	The complement of event A (denoted $\overline{A}\ or\ A^c$) means “**not A**”; it consists of all simple outcomes that are not in A.
Conditional Probability (“given that”)	$\boldsymbol{P(A\|B) = \dfrac{P(A \cap B)}{P(B)}}$ if independent or disjoint: $P(A\|B) = P(A)$ $P(B\|A) = P(B)$	Means the probability of event A given that event B occurred. Is a rephrasing of the Multiplication Rule. P(A|B) is the proportion of elements in B that are ALSO in A.

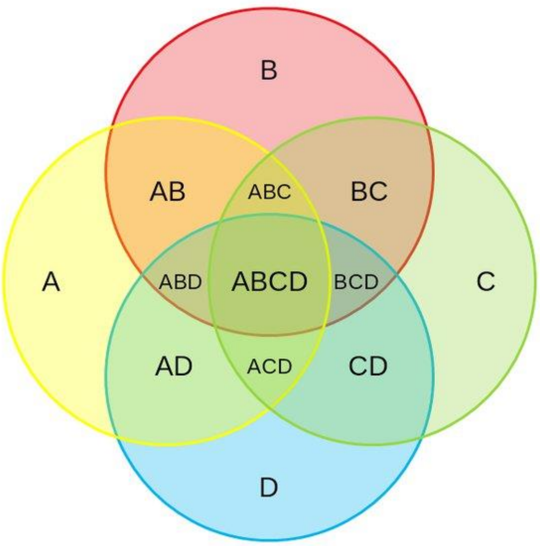
Total Probability Rule	$P(A) = P(A \cap B_1) + \cdots + P(A \cap B_n)$ $= P(B_1)\, P(A\|B_1) + \cdots + P(B_n)\, P(A\|B_n)$ $P(A) = P(A \cap B) + P(A \cap \bar{B})$ $= P(B)\, P(A\|B) + P(\bar{B})\, P(A\|\bar{B})$	To find the probability of event A, partition the sample space into several disjoint events. A must occur along with one and only one of the disjoint events.
Bayes' Theorem	$\boldsymbol{P(A\|B)} = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\, P(B\|A)}{P(B)}$ $= \frac{\boldsymbol{P(A)\, P(B\|A)}}{\boldsymbol{P(A)\, P(B\|A) + P(\bar{A})\, P(B\|\bar{A})}}$	Allows P(A|B) to be calculated from P(B|A). Meaning it allows us to reverse the order of a conditional probability statement, and is the only generally valid method!
De Morgan's Law	$\overline{P(A \cup B)} \equiv \overline{P(A)} \cap \overline{P(B)}$ $\overline{P(A \cap B)} \equiv \overline{P(A)} \cup \overline{P(B)}$	Uses negation to convert an "or" to an "and". Uses negation to convert an "and" to an "or".

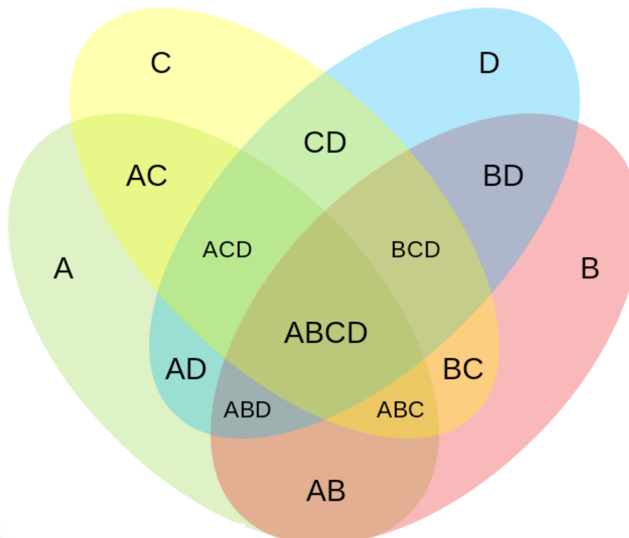
Venn Diagrams

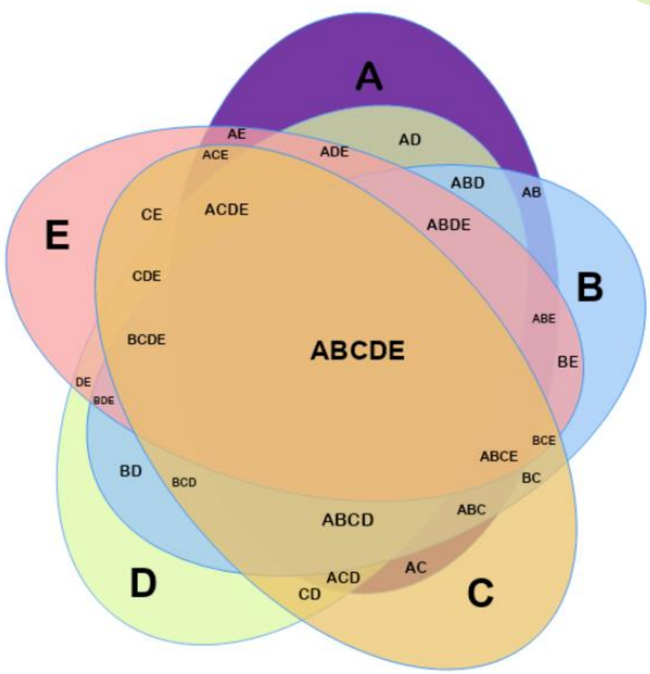



A
AB
B


A
A∩B
B
A∩B∩C
A∩C
B∩C
C


B
AB
ABC
BC
A
ABD
ABCD
BCD
C
AD
ACD
CD
D


C
D
CD
AC
BD
ACD
BCD
A
B
ABCD
AD
BC
ABD
ABC
AB


A
AE
ACE
ADE
AD
ABD
AB
E
CE
ACDE
ABDE
CDE
B
ABE
BCDE
ABCDE
BE
DE
BDE
BCE
ABCE
BC
BD
BCD
ABCD
ABC
AC
ACD
CD
D
C

Sources:

- SNHU MAT 229 - Mathematical Proof and Problem Solving, How To Prove It - A Structured Approach, 3rd Edition - Daniel J. Vellman, Cambridge University Press, 2019.

- SNHU MAT 230 - Discrete Mathematics, zyBooks.

- Learn more about probability distributions here: http://blog.cloudera.com/blog/2015/12/common-probability-distributions-the-data-scientists-crib-sheet/