**Harold’s Series**

**Cheat Sheet**

22 September 2025

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| **Sigma Notation** | | |
|  | | |
| **Sequence** |  | A sequence separates terms with a comma |
| **Series** |  | A series adds up the sequence terms |
| **Finite Series** |  | From or to |
| **Infinite Series** |  | From to ∞ |
| **Convergent** |  | Approaches a constant value |
| **Divergent** |  | Grows to infinity |

Related cheat sheets:

* [Harold’s Infinite Series Cheat Sheet](https://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Infinite_Series_Cheat_Sheet.pdf)
* [Harold’s Infinite Products Cheat Sheet](https://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Infinite_Products_Cheat_Sheet.pdf)
* [Harold’s Series Convergence Tests Cheat Sheet](https://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Series_Convergence_Tests_Cheat_Sheetpdf)

**Arithmetic and Geometric Series**

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| --- | --- | --- |
| **Operation** | **Arithmetic Series** | **Geometric Series** |
| **Summation Notation** |  |  |
| **Summation Expanded** |  |  |
| **Recursive nth Term of Sequence** |  |  |
| **Explicit nth Term of Sequence** |  |  |
| **Sum of n Terms**  **(Finite Series)** |  |  |
| **Sum of Terms**  **(Infinite Series)** |  |  |
| **Archimedes Geometric Series Example** |  | But first, let’s take a look at this visual representation of an ... |
| **Another Geometric Series Example** |  |  |

**Summation Formulas**

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| --- | --- |
| **Type** | **Summation Formulas** |
| **Constant Multiple Rule** |  |
| **Sum Rule** |  |
| **Index Shift** |  |
| **Sum of Powers**  (Arithmetic Series) |  |
| **Interesting Summation Formulas** |  |

**Binomial Theorem**

|  |  |  |  |
| --- | --- | --- | --- |
| **Binomial Series** | | **Expanded** | |
| **Pascal’s Triangle** | 1  1 1  1 2 1  1 3 3 1  1 4 6 4 1  1 5 10 10 5 1 |  |  |
| **Example** |  |  | |
| **Binomial Theorem** |  | | |
|  | |  | |

**Factorials and Constants**

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| --- | --- |
| **Operation** | **Formula** |
| **Termial (Tn)** |  |
| **Factorial** |  |
| **Double Factorial** | (Even n)  (Odd n) |
| **Gamma Function**  (Continuous Factorial) |  |
| **Combination** | *Converges for*  *and all complex r , r ≠ 0, where* |
| **Permutation** |  |
| **Fibonacci Sequence** | **F** = {0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, …}  Recursive:  F0 = 0, F1 = 1  **Fn = Fn-1 + Fn-2**  Explicit: |
| **Fibonacci Numbers vs. Pascal’s Triangle** | The Fibonacci numbers are the sums of the “shallow” diagonals (shown in red) of Pascal’s triangle |
| **Golden Ratio** | ≅ 1.6180 33988 74989 48482 04586 83436 56381 17720 30917 98057 …  Solve for |
| **Fibonacci Numbers vs. Golden Ratio** | Fibonacci Sequence |
| **Euler’s Identity** | Sincein |
| **Euler’s Number** | *e* ≅ 2.71828 18284 59045 23536 02874 71352 66249 77572 47093 69995 .... |
| **Imaginary Unit** |  |
| **Archimedes’ Constant (pi)** | π ≅ 3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510 ... |

**Sources**

* Dawkins, Paul (2023). Section 10 : Series And Sequence, Paul’s Online Notes. <https://tutorial.math.lamar.edu/Classes/CalcII/SeriesIntro.aspx>
* MedCalc (2025). TERMIAL function. <https://www.medcalc.org/manual/termial-function.php>
* Story of Mathematics (2024). What is a geometric series? <https://www.storyofmathematics.com/geometric-series/>
* Wireless Pi (2025). Rapid Skill Acquisition. <https://wirelesspi.com/sdr-course/>