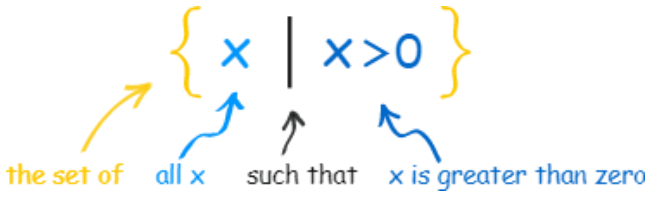
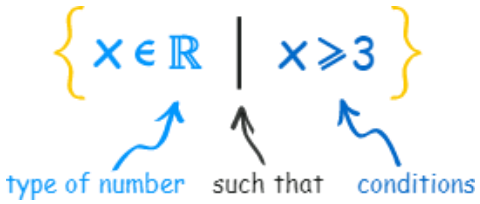
Harold's Sets
Cheat Sheet
5 May 2026

Set Definitions

Term	Definition	Examples
Set	A well-defined collection of distinct mathematical objects	C = {2, 4, 5} denotes a set of three numbers: 2, 4, and 5 D = {(2, 4), (−1, 5)} denotes a set of two ordered pairs of numbers
Element	Objects, members	a, 3, (x, y)
Pair	Ordered pair. An element with two members. Order matters.	(x, y)
Tuple	Ordered tuple. A column of three mathematical objects. Order matters.	(a, b, c) *or* $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$
n-Tuple	Ordered n-tuple. $\mathbb{Z}^3$ is the set of all **3-tuples** whose entries are integers. Order matters.	$\mathbb{Z}^3 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a, b, c \in \mathbb{Z} \right\}$
Set-Builder Notation	Set $Uppercase_letter = \{number_type\ [: or\ |]formula \wedge restrictions\ or\ conditions\}$	$F = \{n \in \mathbb{Z} : n^3 \wedge 1 \leq n \leq 100\}$ The set of cubes of the first 100 positive integers.
Roster Notation	A list of the elements enclosed in curly braces with the individual elements separated by commas	A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Set-Builder Notation





Set-Builder Notation: { x ∈ ℝ | x ≤ 2 or x > 3 }

Number Line:

Interval Notation: (−∞, 2] U (3, +∞)

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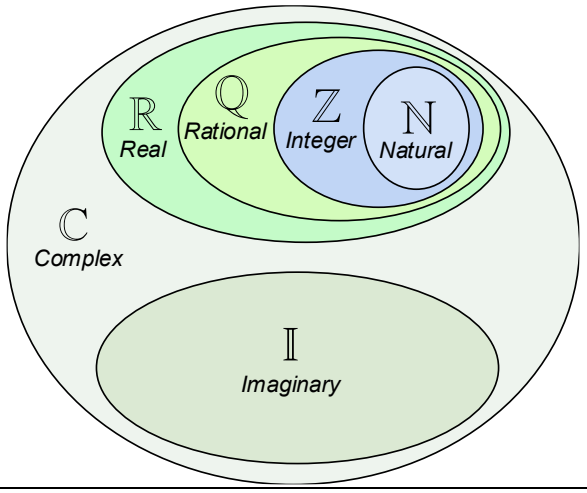
Number Sets

Symbol	Definition	Set Notation	Examples	Equations
$\emptyset$	**Empty** or null set	{ }	$\emptyset \in \{\emptyset\}$	$1 = 2$
$\mathbb{P}$	**Prime** numbers	{a, b ∈ $\mathbb{Z}^+$: (p\ab ⟶ p\a ∨ p\b)}	{2, 3, 5, 7, 11, 13, ...}	$\gcd(n, m) = 1$
$\mathbb{N}$ $\mathbb{N}_0$	**Natural** numbers	{x ∈ $\mathbb{Z}$: x ≥ 0}	{0, 1, 2, 3, 4, ...} (per ISO 80000-2 2-7.1)	$x - 3 = 0$
$\mathbb{W}$	**Whole numbers**	{x ∈ $\mathbb{Z}$: x ≥ 0}	{0, 1, 2, 3, ...}	n ≥ 0
$\mathbb{Z}$	**Integers**	{x : x = ±$\mathbb{N}$ ∨ x = 0}	{..., −3, −2, −1, 0, 1, 2, 3, ...}	$x + 7 = 0$
$\mathbb{Q}$	**Rational** numbers	{p/q : p, q ∈ $\mathbb{Z}$ ∧ q ≠ 0}	{0, ¼, ½, ¾, 1}	$4x - 1 = 0$
$\mathbb{I}$	**Irrational** numbers	{x ∈ $\mathbb{R}$: x ∉ $\mathbb{Q}$}	{0, ¼, ½, ¾, 1}	$4x - 1 = 0$
$\mathbb{A}$	**Algebraic** numbers	{ x ∈ $\mathbb{R}$: x = root of a one-variable polynomial ∧ coefficients ∈ $\mathbb{Q}$}	{5, -7, ½, $\sqrt{2}$}	$2x^2 + 4x - 7 = 0$
$\mathbb{T}$	**Transcendental** numbers	{x ∈ $\mathbb{R}$: x ∉ $\mathbb{A}$, x ∉ $\mathbb{Q}$}	{π, e, e^{π}, sin(x), $\log_b$ a}	$\mathbb{T} = \mathbb{U} - \mathbb{A}$
$\mathbb{R}$	**Real** numbers	{x : x corresponds to a number on the number line}	{ π , 3.1415, -1, ⅞, $\sqrt{2}$}	$x^2 - 2 = 0$
$\mathbb{I}$	**Imaginary** numbers	{b : bi where $i = \sqrt{-1}$}	{2i, $\sqrt{-1}$}	$x^2 + 1 = 0$
$\mathbb{C}$	**Complex** numbers	{a, b ∈ $\mathbb{R}$: a + bi}	{1 + 2i, -3.4i, ⅝}	$x^2 - 4x + 5 = 0$
$\mathbb{M}$	**Matrix**	{A ∈ $\mathbb{M}_n$ ∧ n ∈ $\mathbb{N}$: $n \times n$ matrix}	$\lvert\mathbb{M}_n\rvert = \lvert A\rvert \neq 0$	$\lvert A\rvert = \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix} = 2$
$\mathbb{U}$	**Universal** set	all possible values in a particular context		

$\emptyset \subset \mathbb{P} \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{A} \subset \mathbb{R} \subset \mathbb{C} \subset \mathbb{U}$



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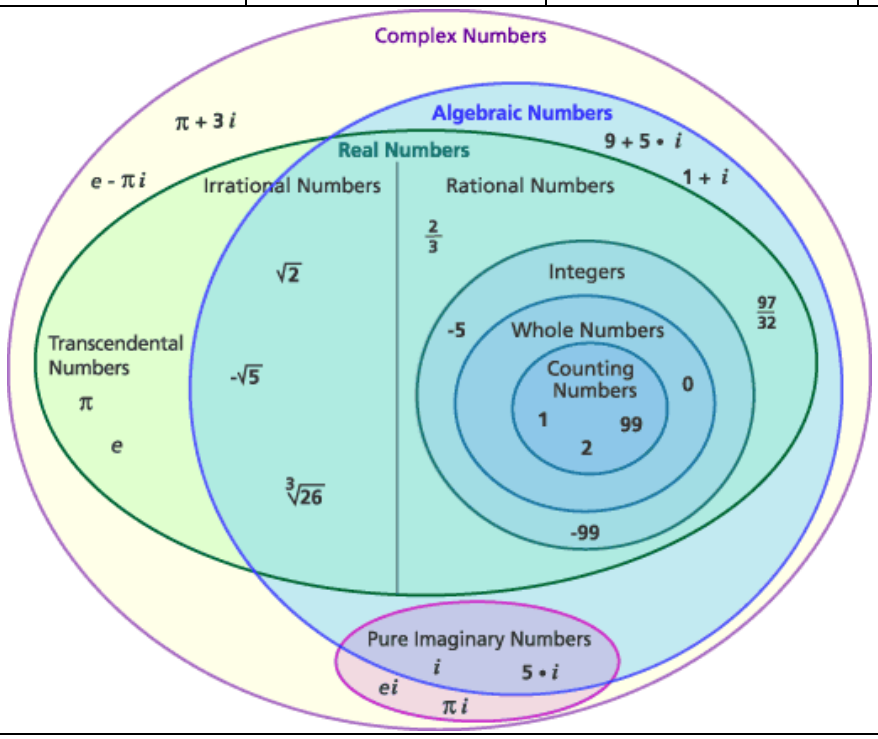
Special Number Sets

Symbol	Definition	Set Notation	Examples	Equations
$\mathbb{N}^*$ $\mathbb{N}_1$	**Non-zero naturals**	$\{x \in \mathbb{N} : x > 0\}$	{1, 2, 3, ...}	n = 1
$\mathbb{Z}^*$ $\mathbb{Z} - \{0\}$	**Non-zero integers**	$\{x \in \mathbb{Z} : x \neq 0\}$	{..., −3, −2, −1, 1, 2, 3, ...}	n ≠ 0
$\mathbb{Z}^+$	**Positive integers**	$\{x \in \mathbb{Z} : x > 0\}$	{1, 2, 3, ...}	n > 0
$\{0\}$	**Zero integer**	$\{x \in \mathbb{Z} : x = 0\}$	{0}	n = 0
$\mathbb{Z}^-$	**Negative integers**	$\{x \in \mathbb{Z} : x < 0\}$	{..., −3, −2, −1}	n < 0
$\mathbb{N}$	**Non-negative integers**	$\{x \in \mathbb{Z} : x \geq 0\}$	{0, 1, 2, 3, ...}	n ≥ 0
$\mathbb{Z}^- \cup \{0\}$	**Non-positive integers**	$\{x \in \mathbb{Z} : x \leq 0\}$	{..., −3, −2, −1, 0}	n ≥ 0
$\{0\}, \mathbb{R}^\times$	**Zero real**	$\{x \in \mathbb{R} : x = 0\}$	{0.0}	x = 0
$\mathbb{R}^*$ $\mathbb{R} - \{0\}$ $\mathbb{R} \setminus \{0\}$	**Non-zero real** numbers	$\{x \in \mathbb{R} : x \neq 0\}$	{-0.001, 0.002}	x ≠ 0
$\mathbb{R}^+$ $(0, \infty)$	**Positive real** numbers	$\{x \in \mathbb{R} : x > 0\}$	{0.0001, 0.0002, ...}	x > 0
$\mathbb{R}^-$ $(-\infty, 0)$	**Negative real** numbers	$\{x \in \mathbb{R} : x < 0\}$	{..., −0.0002, −0.0001}	x < 0
$[0, \infty)$	**Non-negative real** numbers	$\{x \in \mathbb{R} : x \geq 0\}$	{0, 0.0001, 0.0002, ...}	x ≥ 0
$(-\infty, 0]$	**Non-positive real** numbers	$\{x \in \mathbb{R} : x \leq 0\}$	{..., −0.0002, −0.0001, 0}	x ≤ 0



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Set Laws

Law	Union Example	Intersection Example
Idempotent Laws	$A \cup A = A$	$A \cap A = A$
Associative Laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative Laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity Laws	$A \cup \emptyset = A$	$A \cap \mathbb{U} = A$
Domination Laws	$A \cup \mathbb{U} = \mathbb{U}$	$A \cap \emptyset = \emptyset$
Double Complement Law	$(A^c)^c = A$	
Complement Laws	$A \cup A^c = \mathbb{U}$	$A \cap A^c = \emptyset$
Complements of $\mathbb{U}$ and $\emptyset$	$\mathbb{U}^c = \emptyset$	$\emptyset^c = \mathbb{U}$
De Morgan's Laws	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$
Absorption Laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Set Difference Law		$A \setminus B = A \cap B^c$ $A - B = A \cap B^c$

Set Properties

Property	Description	Examples
Composition	Objects may be of various types. A set may contain elements of different varieties.	A = {2, strawberry, monkey}
Order	The order in which the elements are listed is unimportant	A = { 10, 6, 4, 2 }
Duplicates	Repeating an element does not change the set	A = { 2, 2, 4, 6, 10 }
Notation	Typically, capital letters will be used as variables denoting sets, and lowercase letters will be used for elements in the set	A = {a, b}
Range	Every set A	$\emptyset \subseteq A \subseteq \mathbb{U}$
Empty Set	Set with no members.	$\emptyset$ is a subset of every set.

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Set Notation

Term	Definition	Examples
{ } { }	Denotes a **set**	A = {a, e, i, o, u}
| :	'**Such that**' or 'for which'	B = {x| x ∈ ℕ and x ≤ 5 } B = {x: x ∈ ℕ and x ≤ 5 }
⇒ ≡	Is **equivalent** or identical to	(C ∩ E) ⇒ (x ∈ C ∧ x ∈ E)
|A| $n(A)$	The **cardinality** of A, the number of elements in set A	if A = {(1,2), (3,4), (5,6)}, then |A| = 3
A = B	If and only if they have precisely the **same** elements. A is **equal** to b.	if A = {4, 9} and B = {n^2 : n=2 or n=3}, then A = B
A ⊆ B	If and only if every element of A is also an element of B. A is a **subset** of B.	{1, 8, 1107} ⊆ ℕ
A ⊈ B	A is not a **subset** of B. A is **not contained** in B.	{-1, -8, -1107} ⊈ ℕ
A ⊂ B	A is a **proper subset** of B. A is a subset of B that is not equal to B.	{1, 8, 1107} ⊂ ℕ
A ⊄ B	A is not a **proper subset** of B. A is **not contained** in B.	{-1, -8, -1107} ⊄ ℕ
B ⊇ A	If and only if every element of A is in B. B is a **superset** of A.	{1, 8, 1107} ⊆ ℕ
a ∈ A A ∈ B a ∈ A	A is a member of, an **element** of, or in A	¾ ∈ ℚ
a ∉ A	A is not a member of A, is **not** an **element** of A	3.14 ∉ ℤ
A ∩ B A ⋂ B A ∩ B	The set contains elements that are in both A and B. A∩B is the **intersection** of A and B.	if A = {1, 2} and B = {2 ,3}, then A ⋂ B = {2}
A ∪ B A ⋃ B A ∪ B	The set contains elements that are in either A or B or both. A∪B is the **union** of A and B.	if A = {1, 2} and B = {2, 3}, then A ⋃ B = {1, 2, 3}
A \ B A − B	**Set difference**. The set contains elements that are in A **but not** in B. A\B is "A drop B". A−B is "A difference B".	if A = {1, 2} and B = {2, 3}, then A \ B = {1}
A ⊕ B	The **symmetric difference** is the set of elements that are a member of exactly one of A and B, but not both	A ⊕ B = (A - B) ∪ (B - A)
A ⋂ B = ∅	A and B are **disjoint** sets. No elements in common.	A ⋂ B = ∅
A^k	Cartesian product of a set A with itself	A^k = A x A x ... x A k times

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Logical Form of Set Notation

Set Notation	Logical Statement	Description
A	$x \in A$ $\forall x \{x \in A\}$	• Is an element of
$\neg A$	$x \notin A$ $\forall x \{x \notin A\}$	• Is not an element of
$A = B$ $A = B$	$A \longleftrightarrow B$ $\forall x [(x \in A \longrightarrow x \in B) \land (x \in B \longrightarrow x \in A)]$ $A \subseteq B \land B \subseteq A$	• Equal • Equivalence • Iff • $\stackrel{\text{def}}{=}$
$A \neq B$ $A \neq B$	$\forall x (x \in A \land x \notin B)$	• Not equal
$A \subseteq B$	$\forall x (x \in A \longrightarrow x \in B)$ $\forall x \in A (x \in B)$ $x \notin A \setminus B$	• Subset of • $A \cap B = A \longrightarrow A \subseteq B$
$A \not\subseteq B$	$\exists x (x \in A \land x \notin B)$	• Not a subset of
$A \cap B$	$\forall x (x \in A \land x \in B)$	• Intersection
$A \cup B$	$\forall x (x \in A \lor x \in B)$	• Union
$A \setminus B$	$\forall x (x \in A \land x \notin B)$	• Difference • But Not
$A \oplus B$	$\forall x \{ x \in A - B \lor x \in B - A \}$	• Exactly one
$A \longrightarrow B$	$\forall x (x \notin A \lor x \in B)$	• If – Then
$A \cap B = \emptyset$	$\neg\exists x (x \in A \land x \in B)$ $\forall x \neg (x \in A \land x \in B)$ $\forall x (x \notin A \lor x \notin B)$ $\forall x (x \in A \longrightarrow x \notin B)$	• A and B are disjoint, having no elements in common
$\mathcal{F}$	$\{A_i \mid i \in I\}$	• Family of sets
$x \in \bigcap\mathcal{F}$	$\{x \mid \forall A \in \mathcal{F} (x \in A)\}$ $\{x \mid \forall A (A \in \mathcal{F} \longrightarrow x \in A)\}$	• Intersection of family of sets
$x \in \bigcup\mathcal{F}$	$\{x \mid \exists A \in \mathcal{F} (x \in A)\}$ $\{x \mid \exists A (A \in \mathcal{F} \land x \in A)\}$	• Union of a family of sets
$\bigcap\mathcal{F}$	$\bigcap_{i \in I} A_i = \{x \mid \forall i \in I (x \in A_i)\}$ $\bigcap_{i \in I} A_i = A_1 \cap A_2 \cap A_3 \cap A_4 \cap \ldots$	• Intersection of an indexed family of sets
$\bigcup\mathcal{F}$	$\bigcup_{i \in I} A_i = \{x \mid \exists i \in I (x \in A_i)\}$ $\bigcup_{i \in I} A_i = \{x \in I \mid \exists i \in I A (i, x)\}$ $\bigcap_{i \in I} A_i = A_1 \cup A_2 \cup A_3 \cup A_4 \cup \ldots$	• Union of an indexed family of sets
$x \in \wp(A)$	$x \subseteq A$ $\forall y (y \in x \longrightarrow y \in A)$	• Power Set • All subsets of set A, including $\emptyset$ • $\lvert P(A) \rvert = 2^{\lvert A \rvert}$

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Logical Form of Numbers

Definition	Logical Statement	Description
Even	$\exists k \in \mathbb{Z}\ (x = 2k)$ Set $E = \{2k : k \in \mathbb{Z}\}$ $2\mathbb{Z}$	• Definition of Even
Odd	$\exists k \in \mathbb{Z}\ (x = 2k + 1)$ Set $O = \{2k + 1 : k \in \mathbb{Z}\}$	• Definition of Odd
Prime	$\forall a, b \in \mathbb{Z}^+ \mid (p \backslash ab \longrightarrow p \backslash a \ \vee \ p \backslash b)$	• A positive integer $p > 1$ that has no positive integer divisors other than 1 and p itself is prime. • Here \ means "is a divisor of"
Not Prime	$\exists a, b \in \mathbb{Z}^+ (ab = n \wedge a < n \wedge b < n)$	• a and b are factors of n, so not prime
Divides	$x \mid y \longleftrightarrow \exists k \in \mathbb{Z}\ (y = kx)$	• Divisibility • Divides • Divides into • x divides y evenly • $x \mid y$ to mean "x divides y," • $x \nmid y$ means "x does not divide y"
Rational	$r \in \mathbb{R}\ \exists x, y \in \mathbb{Z}\ ((y \neq 0) \wedge (r = x/y)) \longrightarrow r \in \mathbb{Q}$	• Definition of a Rational number • A fraction composed of two integers, but no division by 0
Irrational	$\forall x \mid x \in \mathbb{R} \wedge x \notin \mathbb{Q}$	• Definition of Irrational number

Logical Form of Geometry

Definition	Logical Statement	Description
Line	$\{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = mx + b\}$ $= \{(0, b), (1, m + b), (2, 2m + b), \ldots\}$	• You can think of the graph of the equation as a picture of its truth set! • $\mathbb{R}^1 = \mathbb{R}$
Plane	$\mathbb{R} \times \mathbb{R} = \{(x, y) \mid$ x and y are real numbers$\}$	• These are the coordinates of all the points in the plane • $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$
3D Space	$\mathbb{R}^3 = \{(x, y, z) \mid$ x, y and z are real numbers$\}$	• These are the coordinates of all the points in 3D space • $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$
Spacetime	$\mathbb{R}^4 = \{(x, y, z, t) \mid$ x, y, z and t are real numbers$\}$	• These are the coordinates of all the points in 3D space and 1D time • $\mathbb{R}^4 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$

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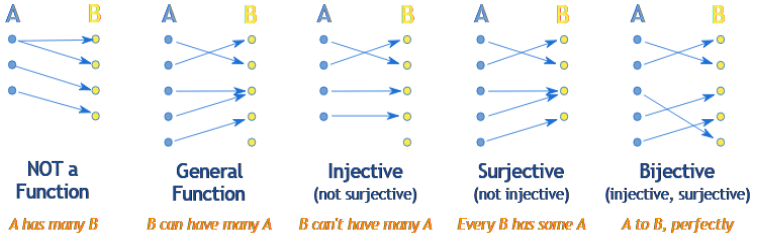
Logical Form of Functions

Definition	Logical Statement	Description
Function	$f: x \longrightarrow y$ $\forall x \in X\ \exists! y \in Y\ ((x, y) \in f)$ $f = \{(a, b) \in A \times B \mid b = f(a)\}$	• General f • f is a relation frrom A to B • Example : $f = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = x^2\}$
Domain	$\mathrm{Dom}(f)$ X	• Domain of f
Range	$\mathrm{Ran}(f)$ $\{f(a) \mid a \in A\}$ Y	• Range ⊆ co-domain • Co-domain • Image of f (linear algebra term)
Injection	$f = \forall y \in Y\ \{\exists$ at **most** one $x \in X$ such that $f(x) = y\}$ $\neg\exists a_1 \in A\ \exists a_2 \in A\ (f(a_1) = f(a_2) \wedge a_1 \neq a_2)$ $\mathbf{\forall a_1, a_2 \in A \mid (f(a_1) = f(a_2) \longrightarrow a_1 = a_2)}$ $f(x) = f(y) \leftrightarrow x = y$ $f(x) \neq f(y) \leftrightarrow x \neq y$	• **One-to-one** • Injective f • For any y there is at most one x • Passes the horizontal line test • Can have orphan y's • *e.g., y is either married or single*
Surjection	$f = \forall y \in Y\ \{\exists$ at **least** one $x \in X$ such that $f(x) = y\}$ $\forall y \in Y, \exists x \in X \mid (f(x) = y)$ Ran(f) = Y	• **Onto** • Surjective f • Every y is mapped to by at least one x • No orphan y's • Entire range of y is covered • *e.g., y is dating at least one x*
Bijection	$f =$ iff $\forall y \in Y\ \{\exists$ a **unique** $x \in X$ such that $f(x) = y\}$ $f(Y) = y \leftrightarrow f^{-1}(y) = Y$	• Bijective = surjective and injective • One-to-one correspondence • Bijective f • Invertible f • Iff has a well-defined inverse (f^{-1}) • Iff both surjective and injective • One-to-one and onto • *e.g., Everyone is married to a spouse*
Inverse	$f^{-1}: B \longrightarrow A$ $\forall b \in B\ \exists! a \in A\ ((b, a) \in f^{-1})$ $f(g(x)) = x$ $f^{-1} \circ f = i_A$ and $f \circ f^{-1} = i_B$	• Inverse f
k-to-1 Correspondence	*Let X and Y be finite sets. The function $f:X \longrightarrow Y$ is a k-to-1 correspondence if, for every $y \in Y$, there are exactly k different $x \in X$ such that $f(x) = y$.*	• Bijection is k = 1



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Cartesian Product

Set Notation	Logical Statement	Description
$A \times B$ $A \times B$ $A \times B$ $A \times B$	$\{(a, b) \mid a \in A \land b \in B\}$ $\{(a, b) \mid a \in A, b \in B\}$	• Cartesian product • Cross product • Set of all ordered pairs in which the first entry is in A and the second entry is in B

Properties of Cartesian Products

Law	Logical Statement	Operator Mnemonic
Distributive	$A \times (B \cup C) = (A \times B) \cap (A \times C)$	• $\times\cap$
	$A \times (B \cup C) = (A \times B) \cup (A \times C)$	• $\times\cup$
Commutative	$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$	• $\times\cap\times$
	$(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$	• $\times\cup\times$
Domination	$A \times \emptyset = \emptyset$ $\emptyset \times A = \emptyset$	• $\times\emptyset$

Relations

Property	i_A	Equivalence (=)	Partial Order (Poset)	Total Order (Linear)
Reflexive	✓	✓	✓	
Symmetric	✓	✓		
Anti-Symmetric			✓	✓
Asymmetric				
Transitive	✓	✓	✓	✓
Total				✓
Density				
Binary Relation		✓		

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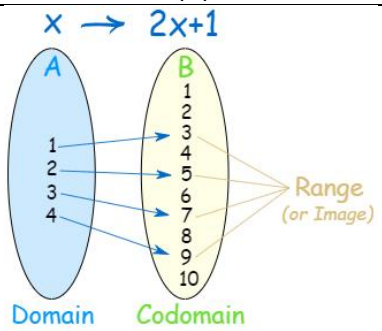
Set Relations (xRy)

Set Notation	Logical Statement	Description
Relation	$R \subseteq A \times B$ $\forall x\ (x \in R \longrightarrow x \in A \times B)$ $R = \{(a, b) \in A \times B \mid \text{conditions}\}$ $xRy = (x, y) \in R$ Example: $D_r = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x \text{ and } y \text{ differ by less than } r\} \Rightarrow \lvert x - y \rvert < r\}$	• Relation from A to B • R is a subset of the cross product
Domain	$Dom(R)$ $\{a \in A \mid \exists b \in B\ ((a, b) \in R)\}$ $Dom(A) \subseteq A$	• The domain of R is the set containing all the first coordinates of its ordered pairs
Codomain (Target)		• All possible values in the range set • Ran(R) is a subset of the Target • The set of <u>possible</u> output values of a function • The definition of a function
Range (Image)	$Ran(R)$ $\{b \in B \mid \exists a \in A\ ((a, b) \in R)\}$ $Ran(B) \subseteq B$	• The range of R is the set containing all the second coordinates of its ordered pairs • The <u>actual</u> or most accurate output values of a function • The image of a function
Inverse (R^{-1})	$\{(y, x) \in Y \times X \mid (x, y) \in R\}$ $(y, x) \in R^{-1} \longleftrightarrow (x, y) \in R$ $(x, y) \in R^{-1} \longrightarrow (x, y) \in R$	• The inverse of R is the relation R^{-1} from B to A with the order of the coordinates of each pair reversed
Composition ($S \circ R$)	$S \circ R = (a, c) \in S \circ R \longleftrightarrow \exists b \mid (a, b) \in R$ and $(b, c) \in S$ $\{(a, c) \in A \times C \mid \exists b \in B\ ((a, b) \in R \text{ and } (b, c) \in S)\}$ $aRb \text{ and } bSc$ $\{(a, c) \in A \times C \mid \exists b \in B\ (aRb \wedge bSc)\}$	• The composition of S and R is the relation $S \circ R$ from A to C • aRb and bSc, meaning R:a ⟶ R:b ⟶ S:b ⟶ S:c, so (R:a, S:c) • Ring operator
Identity (i_A)	$\{(x, y) \in A \times A \mid x = y\}$ $\{(x, x) \mid x \in A\}$	• Identity relation

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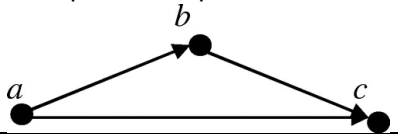
Order Properties of Binary Relations with Two Sets

Property	Logical Statement	Description
Reflexive	xRx $(x, x) \in R$ $\forall x \in A\ (xRx)$ $\forall x \in A\ ((x, x) \in R)$	• $i_A \subseteq R$ where i_A is the identity relation of set A or $i_A = \{(x, x) \mid x \in A\}$ • **Directed graph**: Loop
Anti-Reflexive	$\neg\ (xRx)$ $\forall x \in A\ \neg\ (xRx)$	• **Directed graph**: No loops
Symmetric	$xRy \longrightarrow yRx$ $\forall x \in A\ \forall y \in A\ (xRy \longrightarrow yRx)$	• $R = R^{-1}$ • **Directed graph**: 2-way arrow (edges come in pairs) or no arrows
Anti-Symmetric	$(xRy \wedge yRx) \longrightarrow (x = y)$ $(x \neq y) \longrightarrow \neg\ (xRy) \vee \neg\ (yRx)$ $\forall x \in A\ \forall y \in A\ ((xRy \wedge yRx) \longrightarrow (x = y))$	• Equivalence • **Directed graph**: An arrow from x to y implies that there is no arrow from y to x No:
Asymmetric	$xRy \longrightarrow \neg\ (yRx)$ $\forall x \in A\ \forall y \in A\ \forall z \in A\ (xRy \longrightarrow \neg\ (yRx))$	• Fails the vertical line test, so not a proper function, f(x) • **Directed graph**: 1-way arrow
Transitive	$(xRy \wedge yRz) \longrightarrow xRz$ $\forall x\ \forall y\ \forall z\ ((xRy \wedge yRz) \longrightarrow xRz)$ $\forall x \in A\ \forall y \in A\ \forall z \in A\ ((xRy \wedge yRz) \longrightarrow xRz)$	• $R \circ R \subseteq R$ • Similar to $S \circ R$ • **Directed graph**: Two routes from every vertex A to every vertex B, 1-hop and 2-hops 
Total	$xRy \vee yRx$ $\forall x \in A\ \forall y \in A\ (xRy \vee yRx)$	• Either-or
Density	$xRy \longrightarrow \exists z \mid xRz \wedge zRy$ $\forall x \in A\ \forall y\ (xRy) \longrightarrow \exists z \mid xRz \wedge zRy$	• A middle-man exists
Binary	$R^{-1} \circ R$ = Relation on set A $R \circ R^{-1}$ = Relation on set C	• Relation on set <set> • Binary relation on set <set>
Identity	$i_A = \{(x, y) \in A \times A \mid x = y\}$ $i_A = \{(x, x) \mid x \in A\}$	• Similar to a diagonal matrix

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Mathematical Number Sets → Computer Science Data Types

Symbol	Definition	C Data Type	C++ Data Type
∅	**empty** set, set with no members	void	
ℕ	**natural** numbers	enum unsigned unsigned char unsigned short unsigned int unsigned long unsigned long long	
ℤ	**integers**	char short int long long long	
ℚ	**rational** numbers	NA	std::ratio<1, 10>
ℝ	**real** numbers	float double long double	
𝕀	**imaginary** numbers	(see complex below) double complex z1; im = cimag(z1);	(see complex below) std::complex <double> z1; im = std::imag(z1);
ℂ	**complex** numbers	#include <complex.h> float complex double complex long double complex	#include <complex> std::complex<float> std::complex<double> std::complex<long double>

Sources:

- SNHU MAT 470 - Real Analysis, The Real Numbers and Real Analysis, Ethan D. Bloch, Springer New York, 2011.
- See also “Harold’s Logic Cheat Sheet”.
- https://www.storyofmathematics.com/set-notation
- https://math24.net/set-identities.html

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