**Harold’s Triangles Cheat Sheet**

8 September 2024

**Trig Laws and Formulas**

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| **Law** | **Equation** |
| **Reference Triangle** | Picture |
| **Law of Sines** | $$\frac{sin(A)}{a}=\frac{sin(B)}{b}=\frac{sin(C)}{c}$$ | $$\frac{a}{sin(A)}=\frac{b}{sin(B)}=\frac{c}{sin(C)}$$ |
| **Law of Cosines** | $$a^{2}=b^{2}+c^{2}-2bc∙cos\left(A\right)$$$$b^{2}=a^{2}+c^{2}-2ac∙cos\left(B\right)$$$$c^{2}=a^{2}+b^{2}-2ab∙cos\left(C\right)$$ |
| **Law of Tangents** | $$\frac{a-b}{a+b}=\frac{tan\left[\frac{\left(A-B\right)}{2}\right]}{tan\left[\frac{\left(A+B\right)}{2}\right]}$$$$\frac{b-c}{b+c}=\frac{tan\left[\frac{\left(B-C\right)}{2}\right]}{tan\left[\frac{\left(B+C\right)}{2}\right]}$$$$\frac{a-c}{a+c}=\frac{tan\left[\frac{\left(A-C\right)}{2}\right]}{tan\left[\frac{\left(A+C\right)}{2}\right]}$$$$tan(A)∙tan(B)∙tan(C)=tan(A)+tan(B)+tan(C)$$ |
| **Law of Cotangents** | $$\frac{cot\left(\frac{A}{2}\right)}{s-a}=\frac{cot\left(\frac{B}{2}\right)}{s-b}=\frac{cot\left(\frac{C}{2}\right)}{s-c}$$$$cot(A)∙cot(B)∙cot(C)=cot(A)+cot(B)+cot(C)$$ |
| **Pythagorean Theorem** | If a right triangle, then$$c^{2}=a^{2}+b^{2}$$**Special Case**: Same as Law of Cosines with angle C = 90°.$$c^{2}=a^{2}+b^{2}-2ab∙cos\left(90°\right)$$$$c^{2}=a^{2}+b^{2} -2ab∙0=a^{2}+b^{2} $$ |
| **Sum of Angles** | $$A°+B°+C°=180°$$$$C°=180°-(A°+B°)$$ | $A+B+C=π$ *radians*$$C=π-(A+B)$$ |
| **Area Formula** | $$A=\frac{1}{2}bh=\frac{1}{2}ab∙sin(C)$$ |  |
| **Semi-Perimeter** | $$s=\frac{a+b+c}{2}$$ |
| **Heron’s Formula** | $$A=\sqrt{s(s-a)(s-b)(s-c)}$$ |
| **Mollweide’s Formula** | $$\frac{a+b}{c}=\frac{cos\left[\frac{\left(A-B\right)}{2}\right]}{sin\left(\frac{C}{2}\right)}$$ |

**Solving for Angles, Sides, and Area**

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| **Order of Use** | **Comments** |
| **1. Sum of Angles** | Easiest formula. |
| **2. Pythagorean Theorem** | Use if one of the angles is a right angle (90°). |
| **3. Law of Sines** | Least complex. Use before Law of Cosines, if possible. |
| **4. Law of Cosines** | More complex. Use only once, then use Law of Sines. |
| **5. Heron’s Formula** | Use for area if all three sides are known. |
| **6. Law of Tangents** | Very complex and seldom used. |
| **7. Law of Cotangents** | Most complex and seldom used. |

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| **Given** | **Find** |
| **Angle** | **Side** | **Area** |
| **SSS** | Law of Cosines | Given | Heron’s Formula |
| **SAS** | NA | Law of Cosines | $$A=\frac{1}{2}ab∙sin(C)$$ |
| **SSA** | Law of Sines | NA | $$A=\frac{1}{2}bh$$ |
| **ASS** |
| **SAA** | Sum of Angles | Law of Sines |
| **ASA** |
| **AAS** |
| **AAA** | Given | Unsolvable. Not unique. One side needed. |

**Ambiguous Cases for SSA**

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| **Scenario** | **# of Solutions** | **Illustrations** |
| **SSA** | **0 – 2 Solutions** |  |
| **a < h** | **No Solution** |  |
| **a = h** | **One Solution** |  |
| **b > a > h** | **Two Solutions** |  |
| $$\frac{b}{sin (B)}= \frac{b}{sin (180°-B)}$$ |  |
| **a ≥ b** | **One Oblique Solution** |  |
| When *a = b*, equilateral / isososoclesWhen *a > b*, obtuse |

Source: <https://mathimages.swarthmore.edu/index.php/Ambiguous_Case>

**Interesting Trig Lengths on a Unit Circle**

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| **Trig Graphically** |
| http://www.mrhosek.com/trigonometry/trig.gif | *10 Secret Trig Functions Your Math Teachers Never Taught You - Scientific  American Blog Network* |

**Fixed Angles Triangles**

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| **45-45-90 Triangle** | **30-60-90 Triangle** |
|  | Diagram  Description automatically generated |
| **Proof:**a² + b² = c²x = yx² + x² = 1²2x² = 1x² = ½$$\sqrt{x^{2}}= \sqrt{\frac{1}{2}}$$$$x= \pm \sqrt{\frac{1}{2}}= \pm \frac{1}{\sqrt{2}}= \pm \frac{\sqrt{2}}{2}= \frac{\sqrt{2}}{2}$$ | **Proof:**a² + b² = c²y² + (½)² = 1²y² + ¼ = 1y² = ¾$$\sqrt{y^{2}}= \sqrt{\frac{3}{4}}$$$$y= \pm \sqrt{\frac{3}{4}}= \pm \frac{\sqrt{3}}{\sqrt{4}}= \pm \frac{\sqrt{3}}{2}= \frac{\sqrt{3}}{2}$$ |

**Fixed Sides Triangles**

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| **Pythagorean Triples** |
| A Pythagorean triple is a right triangle with only integer sides.The examples on the right are expressed in the lowest form.You can scale any of these with an integer (e.g., 2,3,4,5) to generate a family of similar triangles.Find the Missing Pieces in Geometry - Ten Steps to Scoring Higher on the GRE - Crash Course for ... | 3 : 4 : 55 : 12 : 137 : 24 : 258 : 15 : 179 : 40 : 4111 : 60 : 61 12 : 35 : 3713 : 84 : 8515 : 112 : 11316 : 63 : 6517 : 144 : 14519 : 180 : 18120 : 21 : 2920 : 99 : 10121 : 220 : 22123 : 264 : 265 | 24 : 143 : 145 28 : 45 : 5328 : 195 : 19732 : 255 : 25733 : 56 : 6536 : 77 : 8539 : 80 : 8944 : 117 : 12548 : 55 : 7351 : 140 : 14952 : 165 : 173 57 : 176 : 18560 : 91 : 10960 : 221 : 22965 : 72 : 9768 : 285 : 293 | 69 : 260 : 26984 : 187 : 20585 : 132 : 15788 : 105 : 13795 : 168 : 19396 : 247 : 265104 : 153 : 185105 : 208 : 233115 : 252 : 277119 : 120 : 169120 : 209 : 241133 : 156 : 205140 : 171 : 221160 : 231 : 281161 : 240 : 289204 : 253 : 325 |
| **Radians and Arc Length** |  |
| Radian = arc length (s) of a unit circles = r $θ$C = $π$ D = $π$ (2r) = 2$π$r**Proof:** If r = 1 (unit circle)then s = $(1)θ = θ$ and C = $2π \left(1\right)=2π$Therefore 360° = 2$π$ radiansTo convert degrees to radians:n° x $\left(\frac{π rad}{180°}\right)$ = m radiansTo convert radians to degrees:m rad x $\left(\frac{180°}{π rad}\right)$ = n° | http://www.mathwarehouse.com/trigonometry/radians/images/picture-s=r-theta-circle.gif |