**Harold’s Trig Proofs**

**Cheat Sheet**

15 March 2023

After proving only three (3) trig formulas, we can easily derive **ALL** the trig formulas!

1. Pythagorean Identity
2. Sum and difference formula for sine
3. Sum and difference formula for cosine

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| **Proof of Pythagorean Identities** | | |
| **Proof** |  | |
| Given | Pythagorean Theorem | |
| Substitute and Simplify |  | |
| Formula |  | |
| **Proof** |  | |
| Given |  | |
| Divide by , then Simplify |  | |
| Formula |  | |
| **Proof** |  |
| Given |  |
| Divide by then Simplify |  |
| Formula |  |

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| **Proof of Sum and Difference Formulas** | | | |
| Trig Sum and Difference Formulas | |  | |
| Proof Diagram | http://www.themathpage.com/atrig/trig_IMG/128.gif | | In VERY simple terms how can I prove the trigonometric identity: Sin (A +  B) = sin A cos B + cos A sin B? - Quora |

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| **Proof of *sin*** |  |
| **Prove Sum** |  |
| Given |  |
| Alternate interior angles are congruent |  |
| Tallest vertical line |  |
| Substitute, then divide and multiply  by AF & FD |  |
| Convert back to trig formulas |  |
| **Prove Difference** |  |
| Replace with |  |
| Simplify |  |
| General Formula [4+5] |  |

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| **Proof of** |  |
| **Prove Sum** |  |
| Given |  |
| Longest horizontal line |  |
| Substitute, then divide and multiply  by AF & DF |  |
| Convert back to trig formulas |  |
| **Prove Difference** |  |
| Replace with |  |
| Simplify |  |
| General Formula [7+8] |  |

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| **Proof of** |  |
| **Prove Sum and Difference** | |
| Given |  |
| Substitute |  |
| Divide by , then Simplify |  |
| General Formula |  |

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| **Proof of Double Angle Formulas (2θ)** | | |
| **Proof** |  | |
| Given |  | |
| Substitute |  | |
| Simplify |  | |
| Formula |  | |
| **Proof** |  | |
| Given |  | |
| Substitute |  | |
| Simplify |  | |
| Formula |  | |
| **Proof** |  |
| Given |  |
| Substitute |  |
| Simplify |  |
| Formula |  |
| **Proof** |  |
| Given |  |
| Substitute |  |
| Simplify |  |
| Formula |  |
| **Proof** |  |
| Given |  |
| Substitute |  |
| Divide by |  |
| Simplify |  |
| Formula |  |

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| **Proof of Half Angle Formulas (θ/2)** | | |
| **Proof** |  | |
| Given |  | |
| Solve for |  | |
| Substitute |  | |
| Solve |  | |
| Formula |  | |
| **Proof** |  | |
| Given |  | |
| Solve for |  | |
| Substitute |  | |
| Solve |  | |
| Formula |  | |
| **Proof** |  |
| Given |  |
| Substitute |  |
| Simplify |  |
| Substitute |  |
| Solve |  |
| Formula |  |

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| **Proof of Cofunction Formulas** | | |
| **Proof** |  | |
| Given |  | |
| Substitute |  | |
| Simplify |  | |
| Formula |  | |
| **Proof** |  | |
| Given |  | |
| Substitute |  | |
| Simplify |  | |
| Formula |  | |
| **Proof** |  |
| Given |  |
| Substitute |  |
| Simplify |  |
| Formula |  |
| **Proof** |  |
| Given |  |
| Substitute |  |
| Simplify |  |
| Formula |  |
| **Proof** |  |
| Given |  |
| Substitute |  |
| Simplify |  |
| Formula |  |
| **Proof** |  |
| Given |  |
| Substitute |  |
| Simplify |  |
| Formula |  |

**Additional Resources**

* AoPSOnline (2025). Art of Problem Solving, Proofs of trig identities. <https://artofproblemsolving.com/wiki/index.php/Proofs_of_trig_identities>