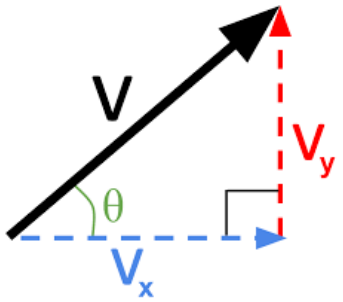
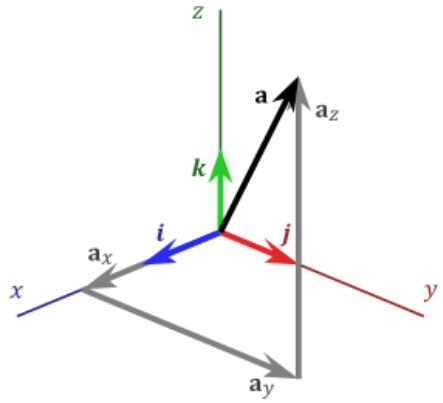
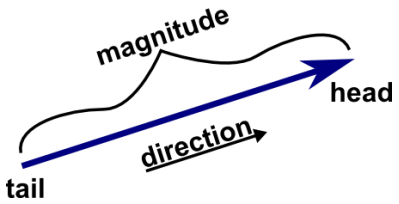
Harold's Vectors
Cheat Sheet
5 May 2026

Definitions

Term	Formula	Example
Vector Notation	$\mathbf{A}, \boldsymbol{a}$	Bold letter
	$\vec{a}, \vec{a}, \vec{a}$	Arrow or harpoon on top
Component Notation	$\boldsymbol{a} = a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\hat{\boldsymbol{k}}$	$\boldsymbol{a} = 3\hat{\boldsymbol{\imath}} + 4\hat{\boldsymbol{\jmath}} + 5\hat{\boldsymbol{k}}$
	$\boldsymbol{a} = \langle a_x, a_y, a_z \rangle$	$\boldsymbol{a} = \langle 3, 4, 5 \rangle$
	$\boldsymbol{a} = r\angle\theta$ (2D)	$\boldsymbol{a} = 5 \angle 53.13°$ (2D)
	 **2D**	 **3D**
2D	*Polar → Rectangular*	*Rectangular → Polar*
	$x = r\cos\theta$ $y = r\sin\theta$ $\tan\theta = \left(\frac{y}{x}\right)$	$r^2 = x^2 + y^2$ $r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$
Vectors Used in Examples	$\boldsymbol{a} = a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\hat{\boldsymbol{k}}$ $\boldsymbol{b} = b_x\hat{\boldsymbol{\imath}} + b_y\hat{\boldsymbol{\jmath}} + b_z\hat{\boldsymbol{k}}$ **2D**: set $a_z = b_z = 0$	$\boldsymbol{a} = 3\hat{\boldsymbol{\imath}} + 4\hat{\boldsymbol{\jmath}} + 5\hat{\boldsymbol{k}}$ $\boldsymbol{b} = 6\hat{\boldsymbol{\imath}} - 7\hat{\boldsymbol{\jmath}} - 8\hat{\boldsymbol{k}}$
Magnitude	$\|\boldsymbol{a}\| = \sqrt{a_x^2 + a_y^2 + a_z^2}$	$\|\boldsymbol{a}\| = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$
	Can also use $\|\boldsymbol{a}\|$. Length of vector, but with no direction (scalar). Similar to a hypotenuse. Think multi-dimensional Pythagorean Theorem.	
Direction	Divided into dimensional components.	A scalar with a direction is a vector. Example: speed vs. velocity
	$\tan\theta = \frac{a_y}{a_x}$	$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{4}{3}\right) \cong 53.13°$

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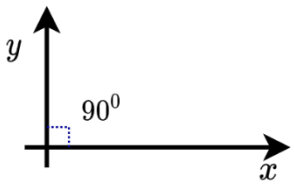
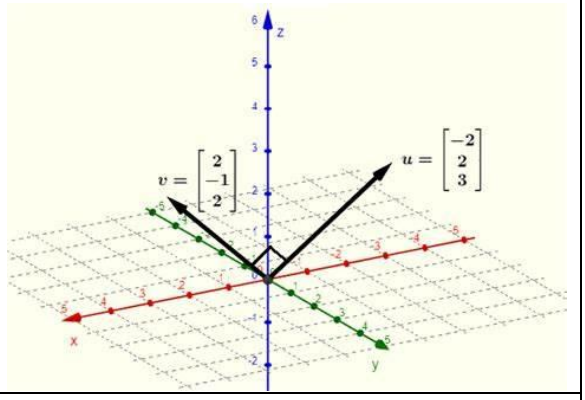
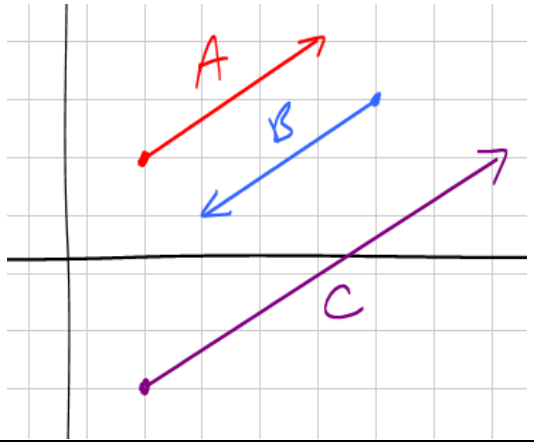
Unit Vector (Basis Vector)	$\hat{\boldsymbol{\imath}} = x - axis = \langle 1, 0, 0 \rangle$ $\hat{\boldsymbol{\jmath}} = y - axis = \langle 0, 1, 0 \rangle$ $\hat{\boldsymbol{k}} = z - axis = \langle 0, 0, 1 \rangle$	Circumflex or "hat" on top. Indicates direction only. Always has a magnitude of one (1 or unit).
	$\vec{\boldsymbol{u}} = \dfrac{\boldsymbol{a}}{\|\boldsymbol{a}\|}$	$\vec{\boldsymbol{u}} = \dfrac{a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\hat{\boldsymbol{k}}}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$
Scalar	*k, m*	A number with no direction or units.
Orthogonal	A change in one dimension does not change in any of the values in the other dimensions.	2D: right angle **Rectangular Coordinates:** The x-axis, y-axis, and z-axis are orthogonal to each other. **Polar Coordinates:** The angle is orthogonal to the line segment length
	if $\boldsymbol{a} \bullet \boldsymbol{b} = 0$	Two vectors are orthogonal if their dot product is zero.
		
Parallel	If $\boldsymbol{a} = k\boldsymbol{b}$	Two vectors are parallel if they have the same direction.
	Collinear but in opposite directions	
Vector vs. Matrix	vector = $1 \times n$ or $n \times 1$ matrix	A matrix with only one (1) row or column.

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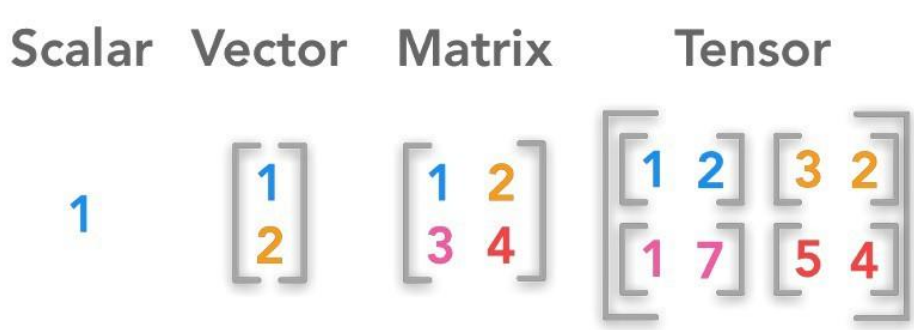
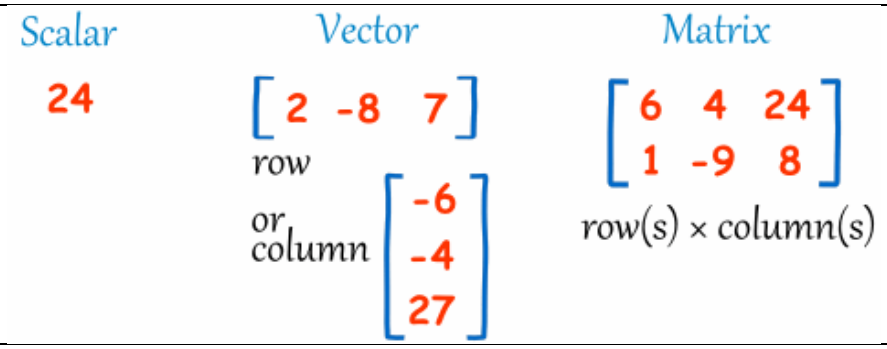
1D – 4D Classification	
Vector Types	

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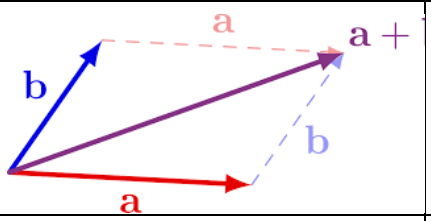
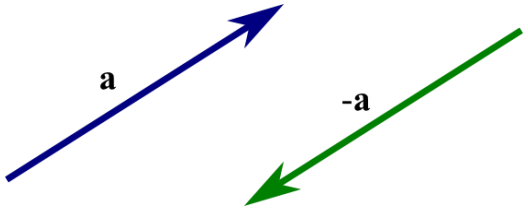
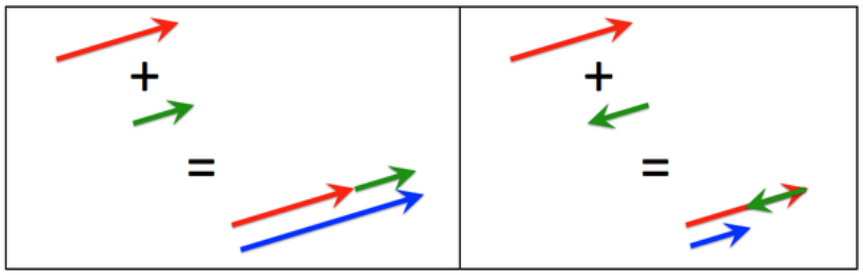
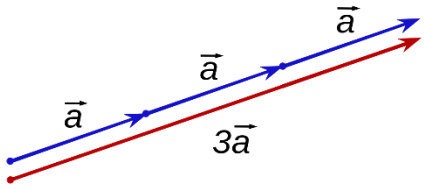
Vector Operations

Operation	Formula	Example
Addition	$\boldsymbol{a}+\boldsymbol{b}=(a_x+b_x)\hat{\boldsymbol{\imath}}+(a_y+b_y)\hat{\boldsymbol{\jmath}}+(a_z+b_z)\hat{\boldsymbol{k}}$	$\boldsymbol{a}+\boldsymbol{b}=(3+6)\hat{\boldsymbol{\imath}}+(4-7)\hat{\boldsymbol{\jmath}}+(5-8)\hat{\boldsymbol{k}}$ $=9\hat{\boldsymbol{\imath}}-3\hat{\boldsymbol{\jmath}}-3\hat{\boldsymbol{k}}$
	$\boldsymbol{a}+\boldsymbol{b}=\boldsymbol{b}+\boldsymbol{a}$	Commutative
	$(\boldsymbol{a}+\boldsymbol{b})+\boldsymbol{c}=\boldsymbol{a}+(\boldsymbol{b}+\boldsymbol{c})$	Associative
	$(k+m)\boldsymbol{a}=k\boldsymbol{a}+m\boldsymbol{a}$	Distributive
	$k(\boldsymbol{a}+\boldsymbol{b})=k\boldsymbol{a}+k\boldsymbol{b}$	
		
Subtraction	$\boldsymbol{a}-\boldsymbol{b}=\boldsymbol{a}+(-\boldsymbol{b})$	Change the direction of $\vec{b}$ then add.
		
Scalar Multiplication	$k\boldsymbol{a}=ka_x\hat{\boldsymbol{\imath}}+ka_y\hat{\boldsymbol{\jmath}}+ka_z\hat{\boldsymbol{k}}$	$3\cdot\boldsymbol{a}=3\cdot3\hat{\boldsymbol{\imath}}+3\cdot4\hat{\boldsymbol{\jmath}}+3\cdot5\hat{\boldsymbol{k}}$ $=9\hat{\boldsymbol{\imath}}+12\hat{\boldsymbol{\jmath}}+15\hat{\boldsymbol{k}}$
	Changes the magnitude only.	
	$k(\boldsymbol{a}+\boldsymbol{b})=k\boldsymbol{a}+k\boldsymbol{b}$	
Dot Product (Scalar Product)	$\boldsymbol{a}\bullet\boldsymbol{b}=a_xb_x+a_yb_y+a_zb_z$	$\boldsymbol{a}\bullet\boldsymbol{b}=(3)(6)+(4)(-7)+(5)(-8)$ $=-50$
	$\boldsymbol{a}\bullet\boldsymbol{b}=\|\|\boldsymbol{a}\|\|\,\|\|\boldsymbol{b}\|\|\cos\theta$	$\cos\theta=\dfrac{\boldsymbol{a}\bullet\boldsymbol{b}}{\|\|\boldsymbol{a}\|\|\,\|\|\boldsymbol{b}\|\|}$
	$\boldsymbol{a}\bullet\boldsymbol{b}=\boldsymbol{b}\bullet\boldsymbol{a}$	Commutative
	$\boldsymbol{a}\bullet(\boldsymbol{b}+\boldsymbol{c})=\boldsymbol{a}\bullet\boldsymbol{b}+\boldsymbol{a}\bullet\boldsymbol{c}$	Distributive
	$k(\boldsymbol{a}\bullet\boldsymbol{b})=k\boldsymbol{a}\bullet\boldsymbol{b}=\boldsymbol{a}\bullet k\boldsymbol{b}$	Scalar Multiplication
	$0\bullet\boldsymbol{u}=0$	Zero Vector Dot Product
	$\boldsymbol{u}\bullet\boldsymbol{u}=\|\|\boldsymbol{u}\|\|^2$	Dot Product and Vector Magnitude Relationship
	Is always a scalar.	

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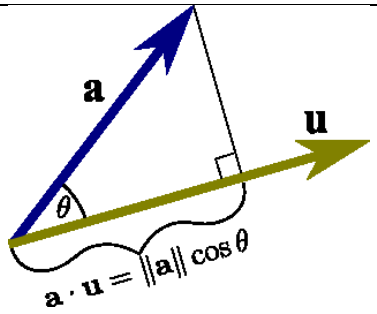
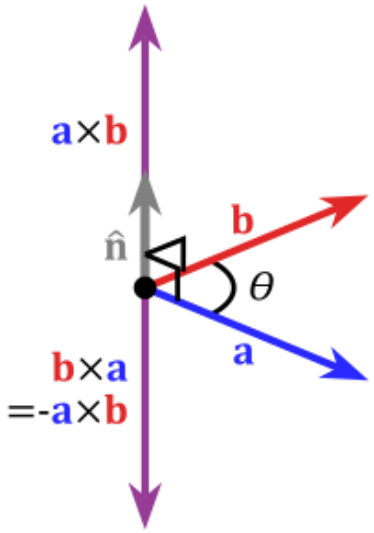
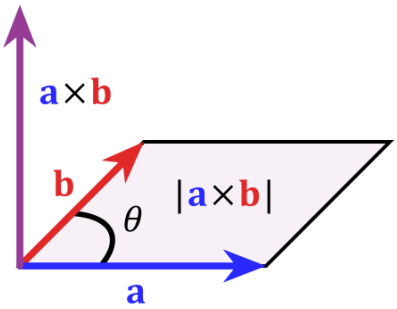
<table>
<tr><td></td><td></td><td> 
</td></tr>
<tr><td rowspan="11">Cross Product
(Vector Product)</td><td colspan="2">$$\boldsymbol{a}\times\boldsymbol{b}=\begin{vmatrix}\boldsymbol{i}&\boldsymbol{j}&\boldsymbol{k}\\a_x&a_y&a_z\\b_x&b_y&b_z\end{vmatrix}=\begin{vmatrix}a_y&a_z\\b_y&b_z\end{vmatrix}\boldsymbol{i}-\begin{vmatrix}a_x&a_z\\b_x&b_z\end{vmatrix}\boldsymbol{j}+\begin{vmatrix}a_x&a_y\\b_x&b_y\end{vmatrix}\boldsymbol{k}$$
$$=\left(a_yb_z-b_ya_z\right)\boldsymbol{i}-\left(a_xb_z-b_xa_z\right)\boldsymbol{j}+\left(a_xb_y-b_xa_y\right)\boldsymbol{k}$$</td></tr>
<tr><td>$\|\boldsymbol{a}\times\boldsymbol{b}\|=\|\boldsymbol{a}\|\,\|\boldsymbol{b}\|\sin\theta$</td><td>$\sin\theta=\dfrac{\|\boldsymbol{a}\times\boldsymbol{b}\|}{\|\boldsymbol{a}\|\,\|\boldsymbol{b}\|}$</td></tr>
<tr><td>$\boldsymbol{a}\times\boldsymbol{b}=-\boldsymbol{b}\times\boldsymbol{a}$</td><td>Anti-Commutative</td></tr>
<tr><td>$(\boldsymbol{a}\times\boldsymbol{b})\times\boldsymbol{c}\neq\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})$</td><td rowspan="2">Not Commutative</td></tr>
<tr><td>$(\boldsymbol{a}\times\boldsymbol{b})\times\boldsymbol{c}=(\boldsymbol{a}\bullet\boldsymbol{c})\boldsymbol{b}-(\boldsymbol{a}\bullet\boldsymbol{b})\,\boldsymbol{c}$</td></tr>
<tr><td>$\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})\neq(\boldsymbol{a}\times\boldsymbol{b})\times\boldsymbol{c}$</td><td rowspan="2">Not Associative</td></tr>
<tr><td>$\boldsymbol{a}\bullet(\boldsymbol{b}\times\boldsymbol{c})=(\boldsymbol{a}\times\boldsymbol{b})\bullet\boldsymbol{c}$</td></tr>
<tr><td>$\boldsymbol{a}\times(\boldsymbol{b}+\boldsymbol{c})=\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{a}\times\boldsymbol{c}$</td><td rowspan="2">Distributive</td></tr>
<tr><td>$(\boldsymbol{a}+\boldsymbol{b})\times\boldsymbol{c}=\boldsymbol{a}\times\boldsymbol{c}+\boldsymbol{b}\times\boldsymbol{c}$</td></tr>
<tr><td>$k(\boldsymbol{a}\times\boldsymbol{b})=k\boldsymbol{a}\times\boldsymbol{b}=\boldsymbol{a}\times k\boldsymbol{b}$</td><td rowspan="2">Scalar Multiplication</td></tr>
<tr><td>$(k\boldsymbol{a})\times\boldsymbol{b}=k(\boldsymbol{a}\times\boldsymbol{b})$
$=\boldsymbol{a}\times(k\boldsymbol{b})$</td></tr>
<tr><td colspan="2">Is always a vector orthogonal to the other two vectors.</td></tr>
<tr><td></td><td> 
</td><td> 
</td></tr>
<tr><td>Scalar Triple Product</td><td colspan="2">$$(\boldsymbol{a}\times\boldsymbol{b})\bullet\boldsymbol{c}=\boldsymbol{a}\bullet(\boldsymbol{b}\times\boldsymbol{c})=\boldsymbol{b}\bullet(\boldsymbol{c}\times\boldsymbol{a})=\boldsymbol{c}\bullet(\boldsymbol{a}\times\boldsymbol{b})=\begin{vmatrix}a_x&a_y&a_z\\b_x&b_y&b_z\\c_x&c_y&c_z\end{vmatrix}$$</td></tr>
<tr><td>Vector Triple Product</td><td colspan="2">$$\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})=(\boldsymbol{a}\bullet\boldsymbol{c})\,\boldsymbol{b}-(\boldsymbol{a}\bullet\boldsymbol{b})\,\boldsymbol{c}$$</td></tr>
</table>

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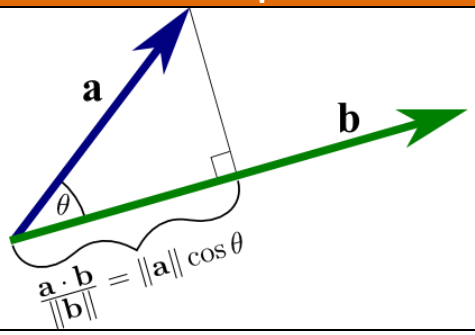
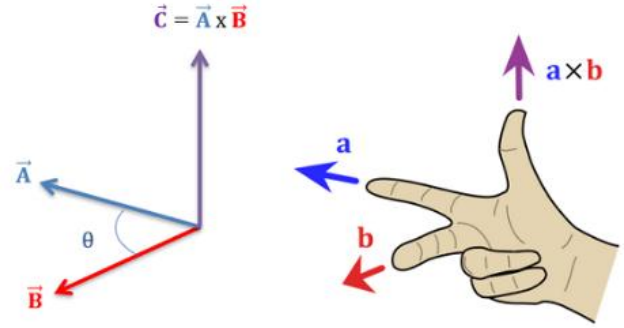
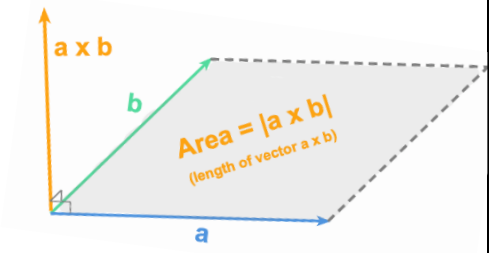
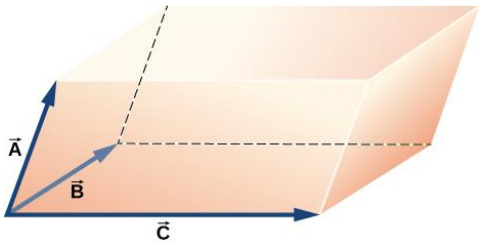
Vector Applications

Application	Formula	Example
Projection	$proj_{\boldsymbol{a}}\boldsymbol{b} = \left(\frac{\boldsymbol{a}\cdot\boldsymbol{b}}{\boldsymbol{a}\cdot\boldsymbol{a}}\right)\boldsymbol{a}$	
Right Hand Rule	The cross product produces a vector orthogonal to the other two vectors.	Use the right hand rule to determine direction of the cross product vector.
		
Area (Parallelagram)	$A = \|\boldsymbol{a} \times \boldsymbol{b}\|$	
Volume (Parallelepiped)	$V = \|(\boldsymbol{a} \times \boldsymbol{b}) \bullet \boldsymbol{c}\|$	
Torque	$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$	$\|\boldsymbol{\tau}\| = r\,F\sin\theta$
Coplanar	Three vectors are coplanar if $(\boldsymbol{a} \times \boldsymbol{b}) \bullet \boldsymbol{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = 0$	All three vectors are in the same plane.

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